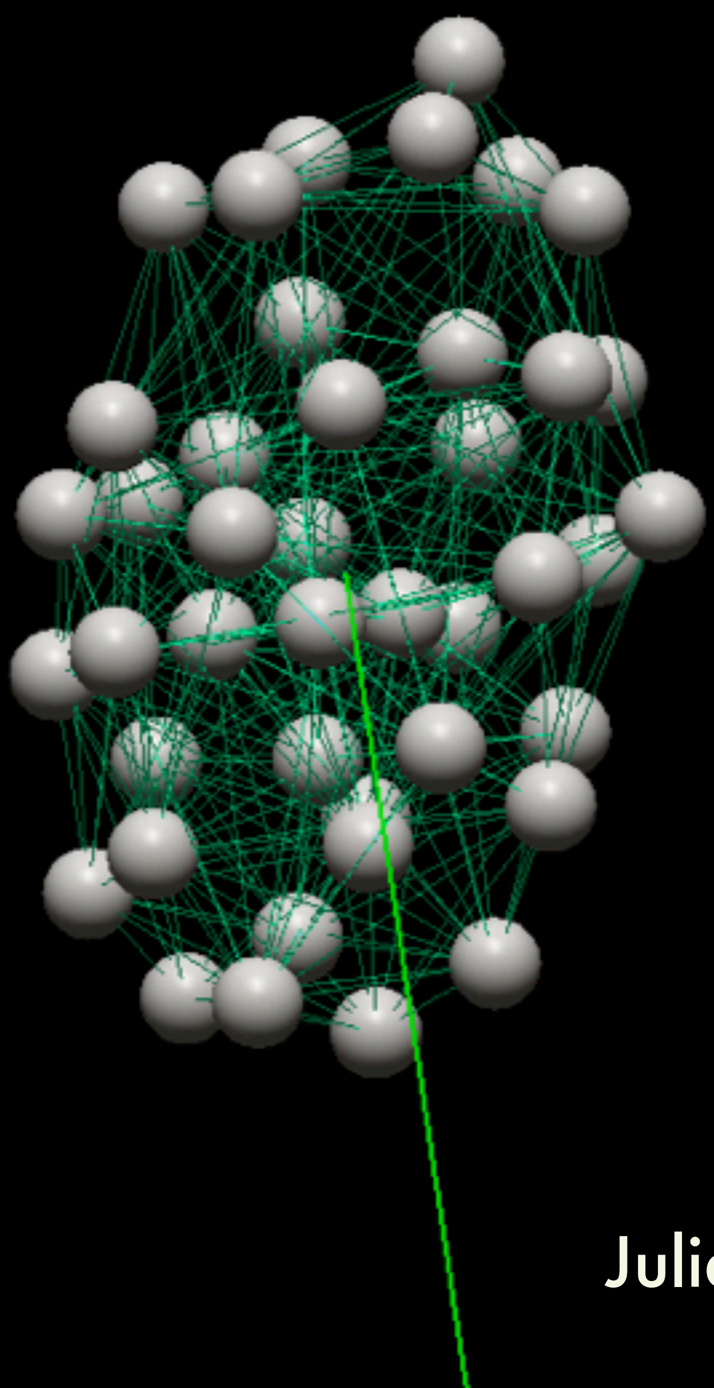
Astro-Visco-Elasto-Dynamics

aka soft astronomy



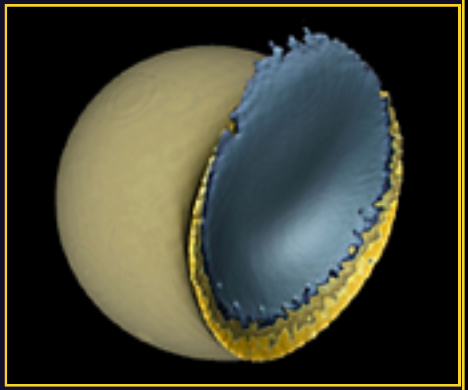
Alice Quillen (U Rochester)

Fiona Nichols-Fleming, Genn Schroeder (UR),
John Shaw, David Gianella(UR) ,
Santiago Loane (UR), Benoît Noyelles (Namur),
Andrea Kueter-Young (Siena),
Yuan-Yuan Chen, Yuhui Zhao(Purple Mountain)
Cindy Ebinger (Tulane), Darin Ragozzine (BYU),
Julien Frouard, Michael Efroimsky (Naval Observatory)

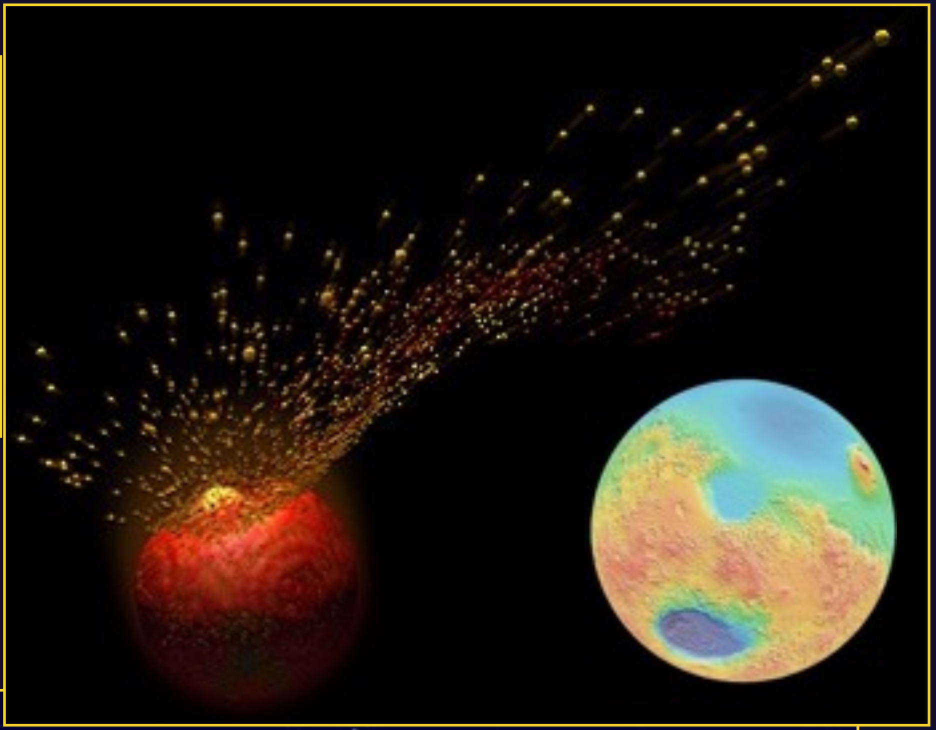
Giant Impacts in the Solar system

Formation of Moon: account for lack of high density core, similar composition of Earth and Moon (e.g., Canup+12)

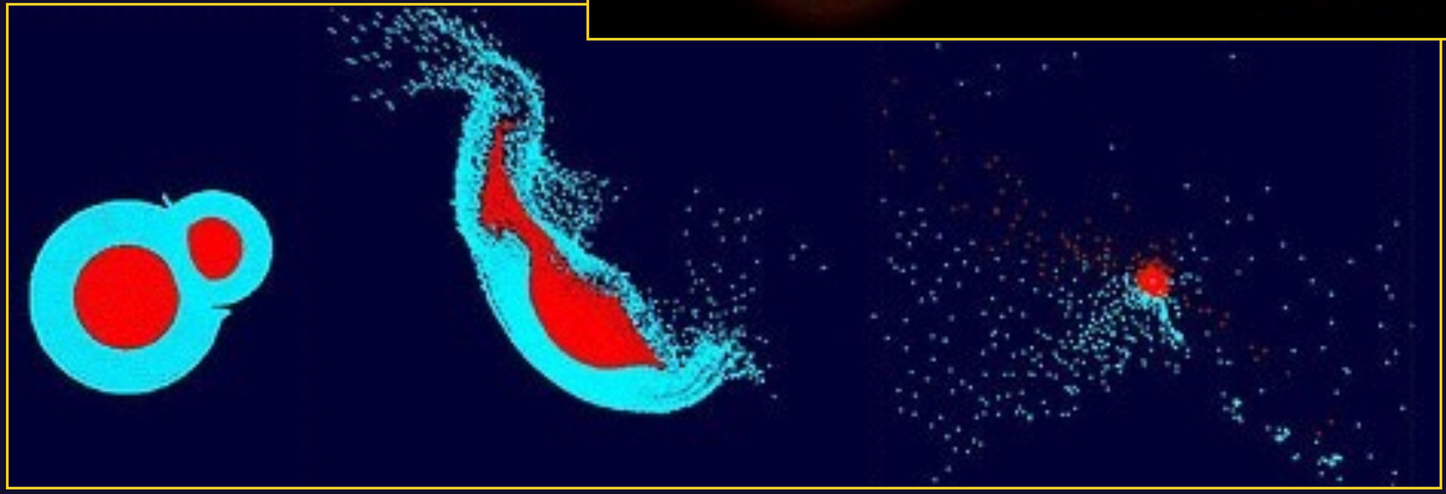
Slow collision with second moon resurfaces one hemisphere (Jutzi & Asphaug)



Giant impact model explaining the crustal dichotomy of Mars (Marinova et al. 2008)



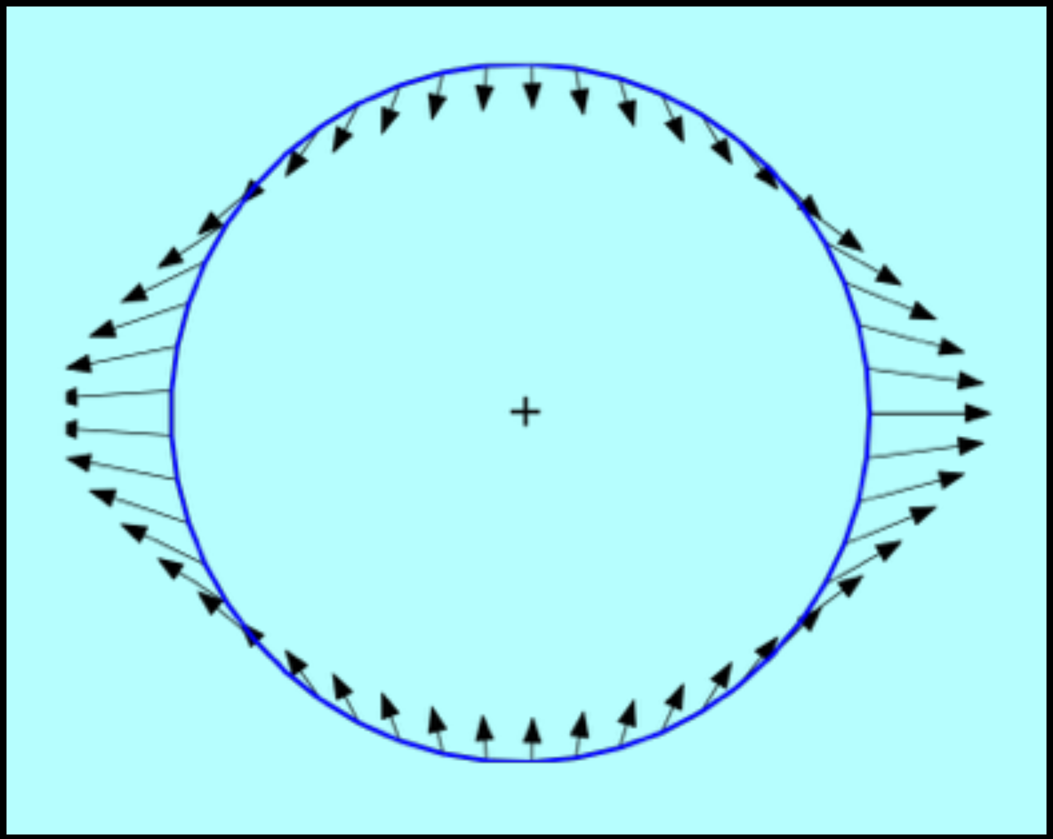
Grazing impact to account for high density of Mercury (Horner)



Collisions compress
(with explosions and
shock waves)

Grazing collisions
make round craters

Tides pull

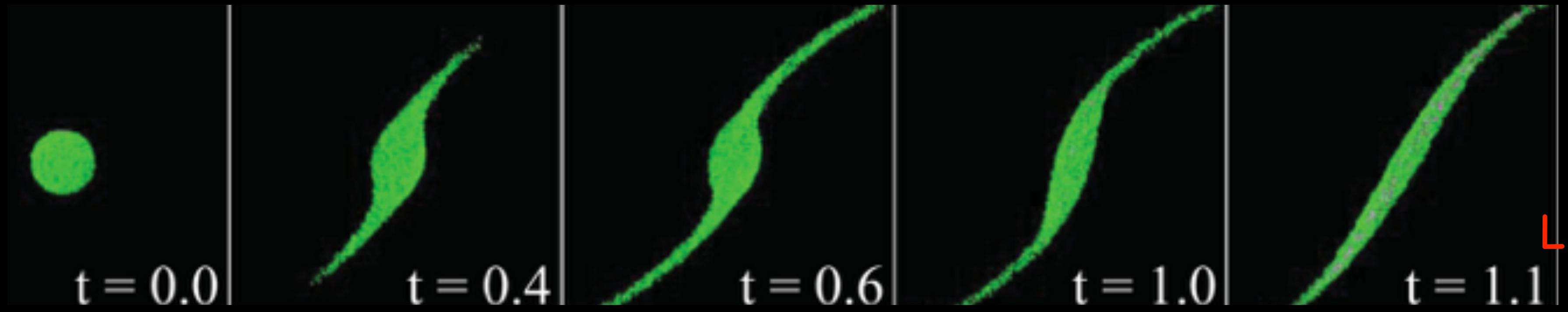


Tidal disruption

Binary asteroid formation

Kevin Walsh

Bars in planetary rings



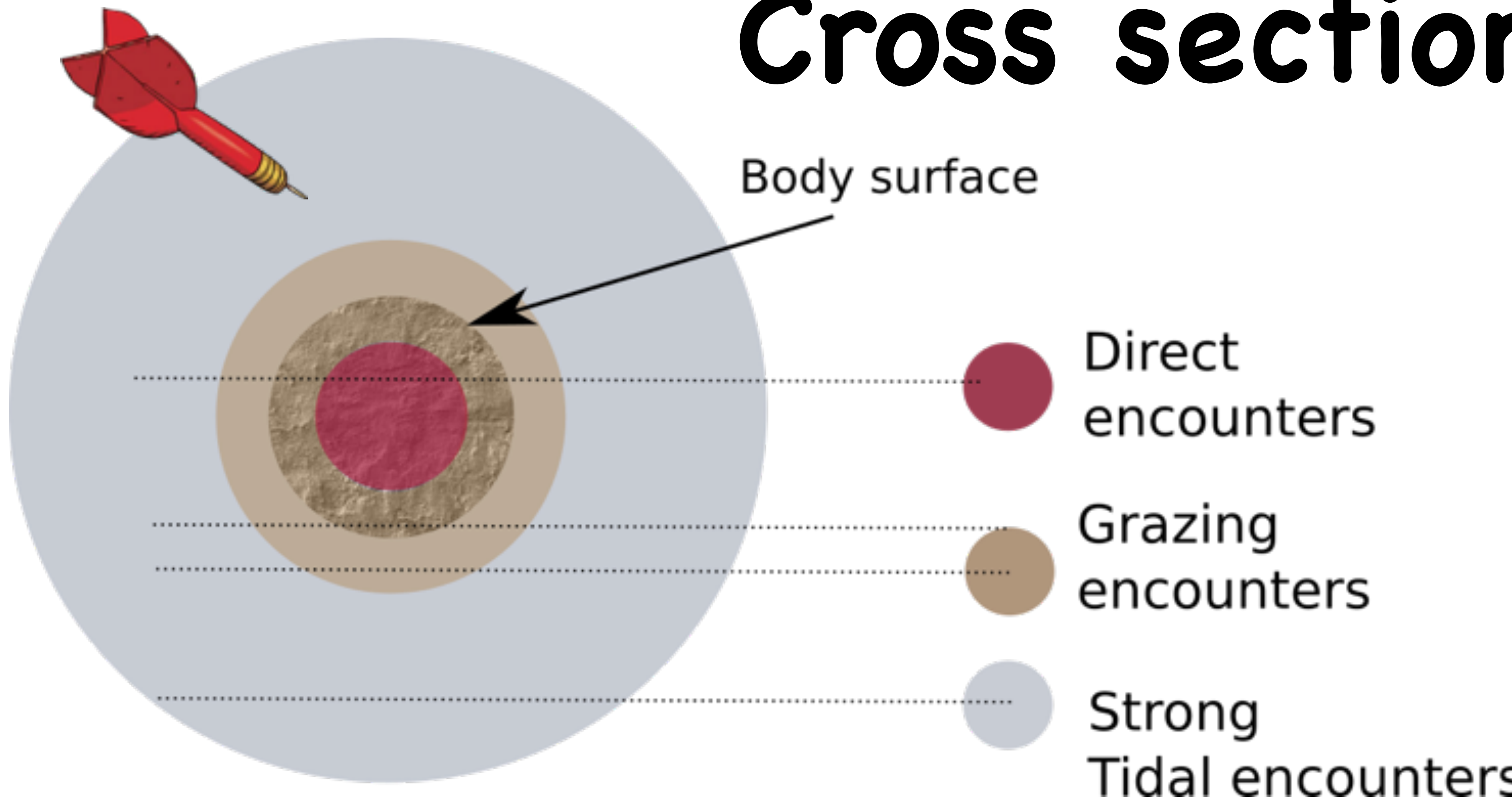
Leinhardt+

Comet disruption

Richardson+1998

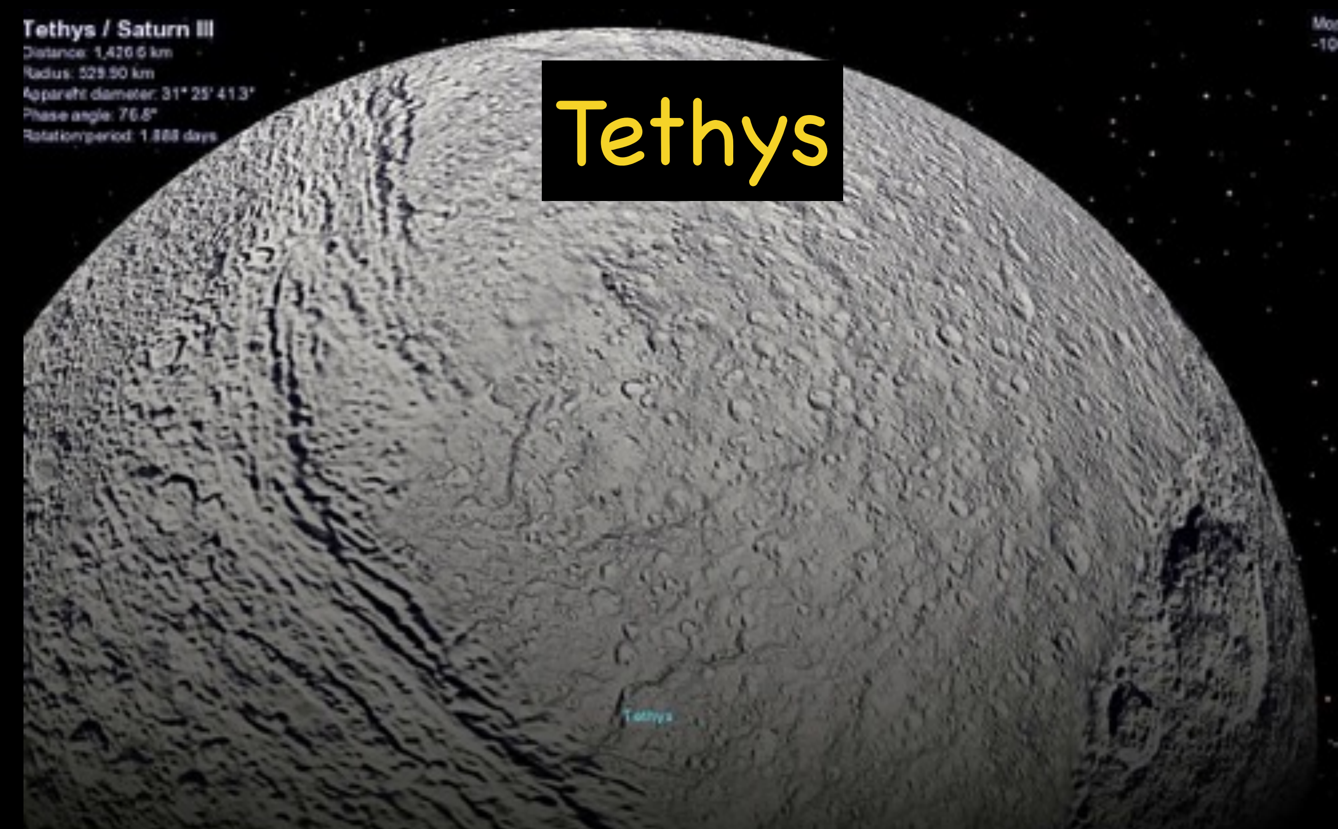
Shoemaker Levy 9

Cross sections

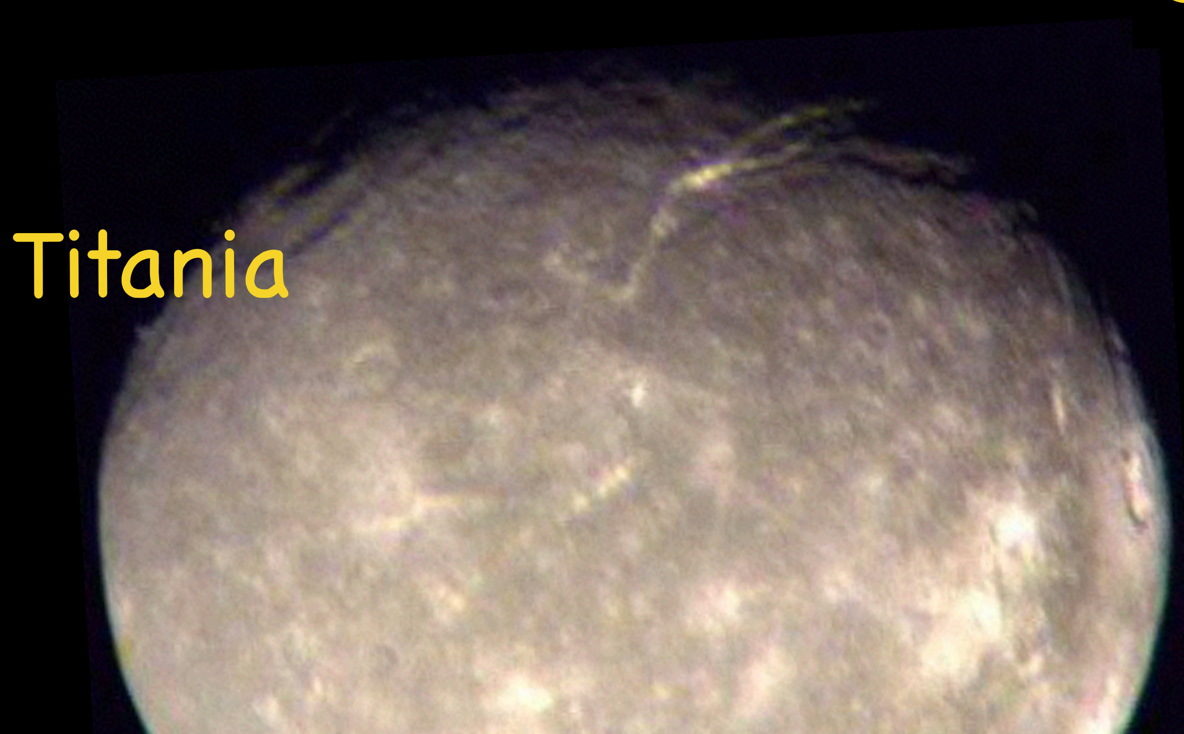


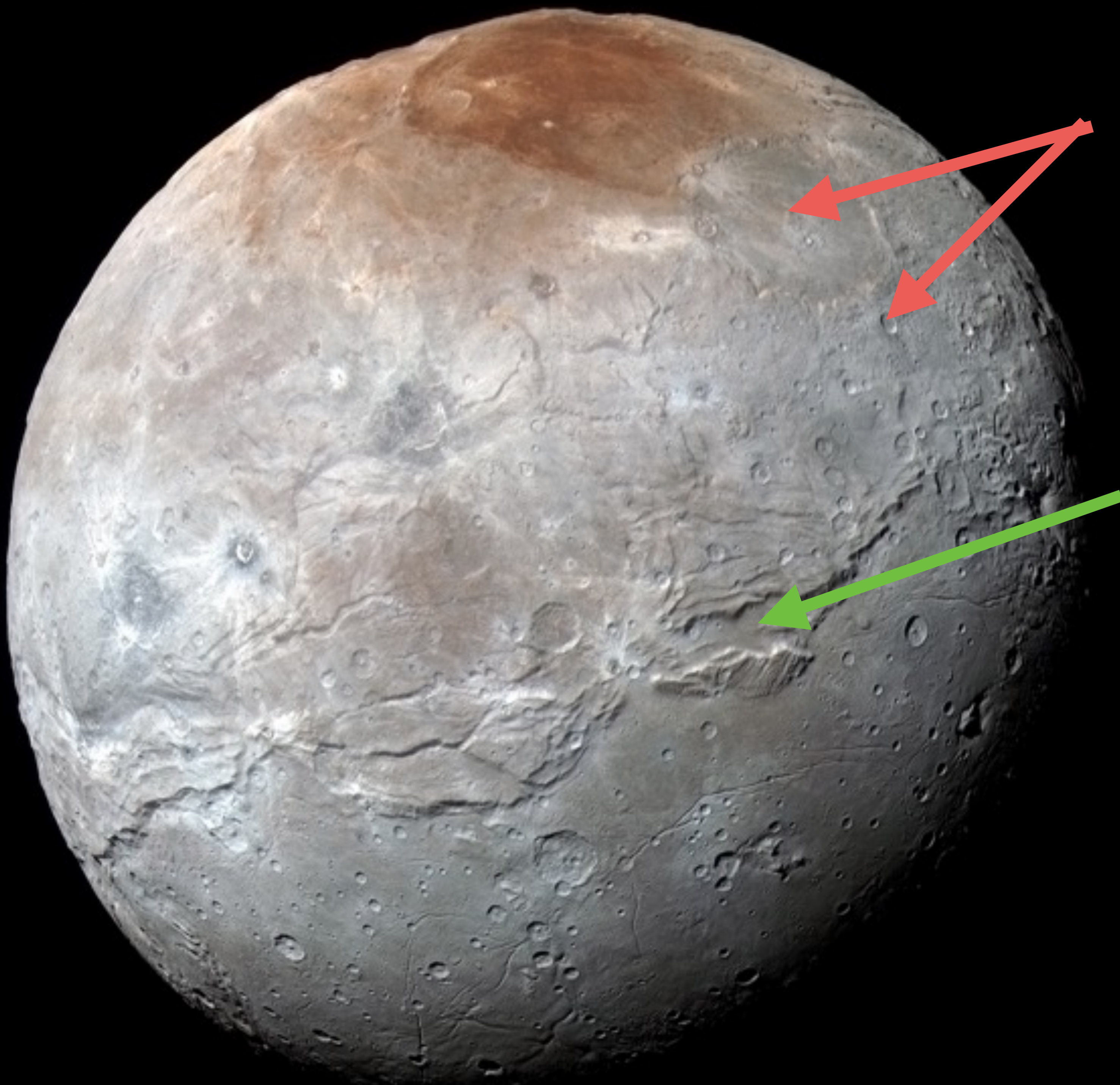
- There were collisions between massive bodies in the early solar system. Grazing collisions more common than direct.
- Strong (nearly grazing) tidal encounters between massive bodies were more common than collisions

Grazing encounters are good at stripping envelope/mantle (Kepler 36 exo-planets density diversity)



Are there surface features on icy bodies that could have been caused by strong tidal encounters?





Crater: formed in minutes —> Astronomical

Graben/rift: formed tectonically over millions of years —> Geophysical

?

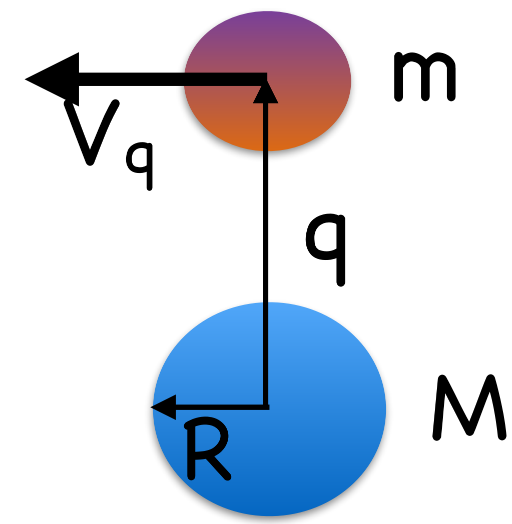
Charon

Barringer Crater
Arizona

On Earth: so
geological?

Great African Rift valley
Diverging continental plates
Tectonic

Deformation Caused by a Tidal Encounter



$$F_t \sim \frac{GmR}{q^3}$$

Tidal force on surface, at radius **R**, caused by mass **m**, at pericenter **q**

$$\Delta v \sim F_t t_{encounter} \sim \frac{2GmR}{q^2 V_q}$$

Velocity kick: V_q is pericenter velocity

$$(\Delta v)^2 \sim \frac{\epsilon^2 E}{\rho}$$

Elastic response: E Young's modulus ρ density, ϵ strain (unitless)

strain

$$\epsilon \sim \left(\frac{e_g}{E}\right)^{\frac{1}{2}} \left(\frac{R}{q}\right)^2 \left(\frac{m}{M}\right)$$

e_g Gravitational binding energy

M mass of primary body

binding energy divided by elastic modulus

radius divided by pericenter

mass ratio

Regime for Crustal Failure

strain

$$\epsilon \sim \left(\frac{e_g}{E}\right)^{\frac{1}{2}} \left(\frac{R}{q}\right)^2 \left(\frac{m}{M}\right)$$

Young's modulus of ice E~1-10 GPa
e_g for bodies like Dione ~1 GPa

binding energy $$e_g = \frac{GM^2}{R^4} = 1.2\text{GPa} \left(\frac{R}{1000\text{ km}}\right)^2 \left(\frac{\rho}{1\text{g cm}^{-3}}\right)$$

Ice fails under tensile stress of 1-10MPa; strains **ε ~ 0.003**

➡ Strong tidal encounters can induce tensile stresses large enough for brittle crustal failure

$$t_{grav} = \sqrt{\frac{R^3}{GM}} = \sqrt{\frac{3}{4\pi G\rho}} = 2000\text{s} \left(\frac{\rho}{1\text{ g cm}^{-3}}\right)^{-\frac{1}{2}}$$ =30 minutes

➡ Strong tidal encounters are **high strain rate**

$\varepsilon/t_{grav} \sim 10^{-6}\ s^{-1}$

On short timescales

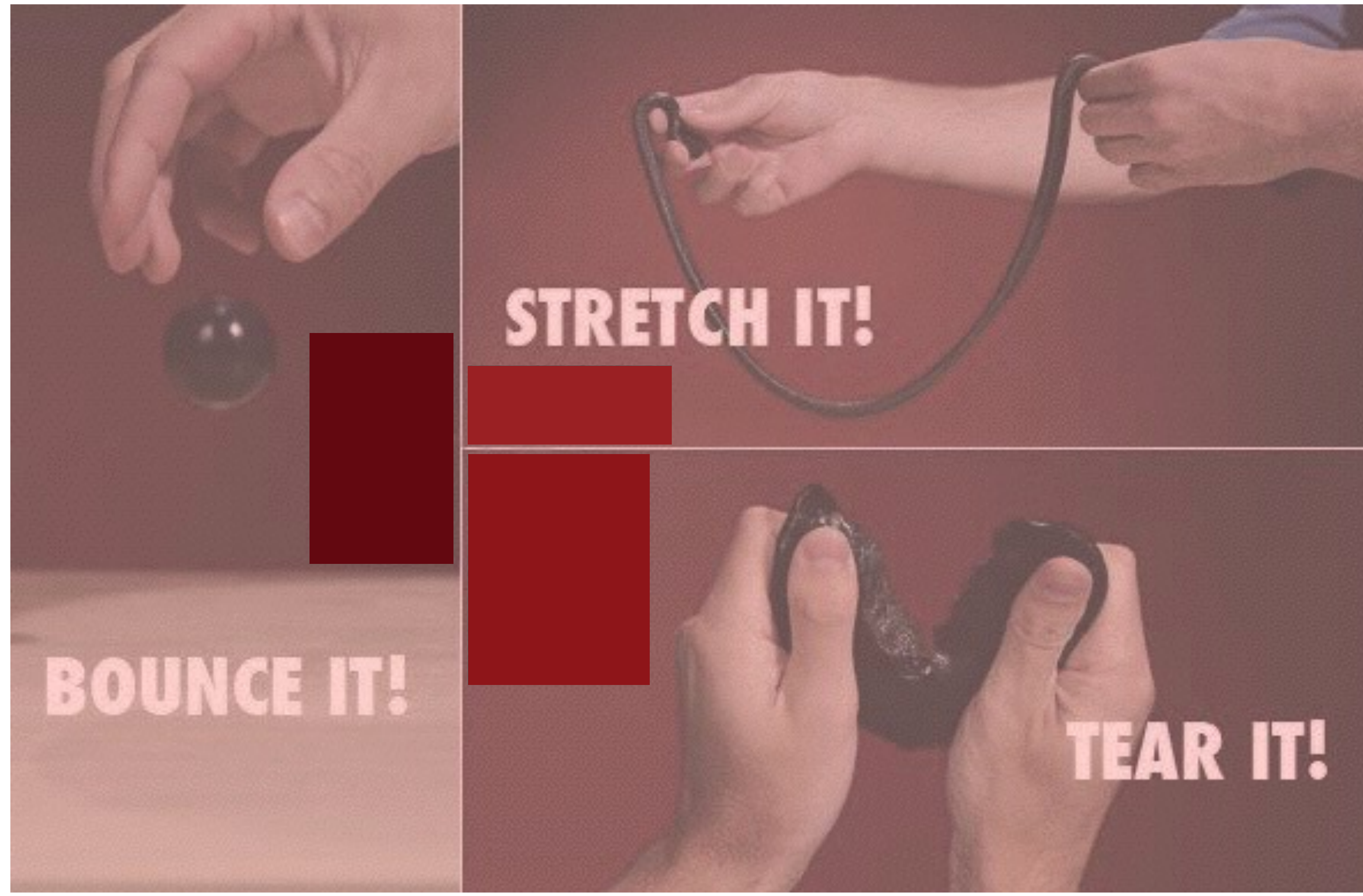
interior is elastic

Tidal encounters
(done in hours)

On LONG timescales

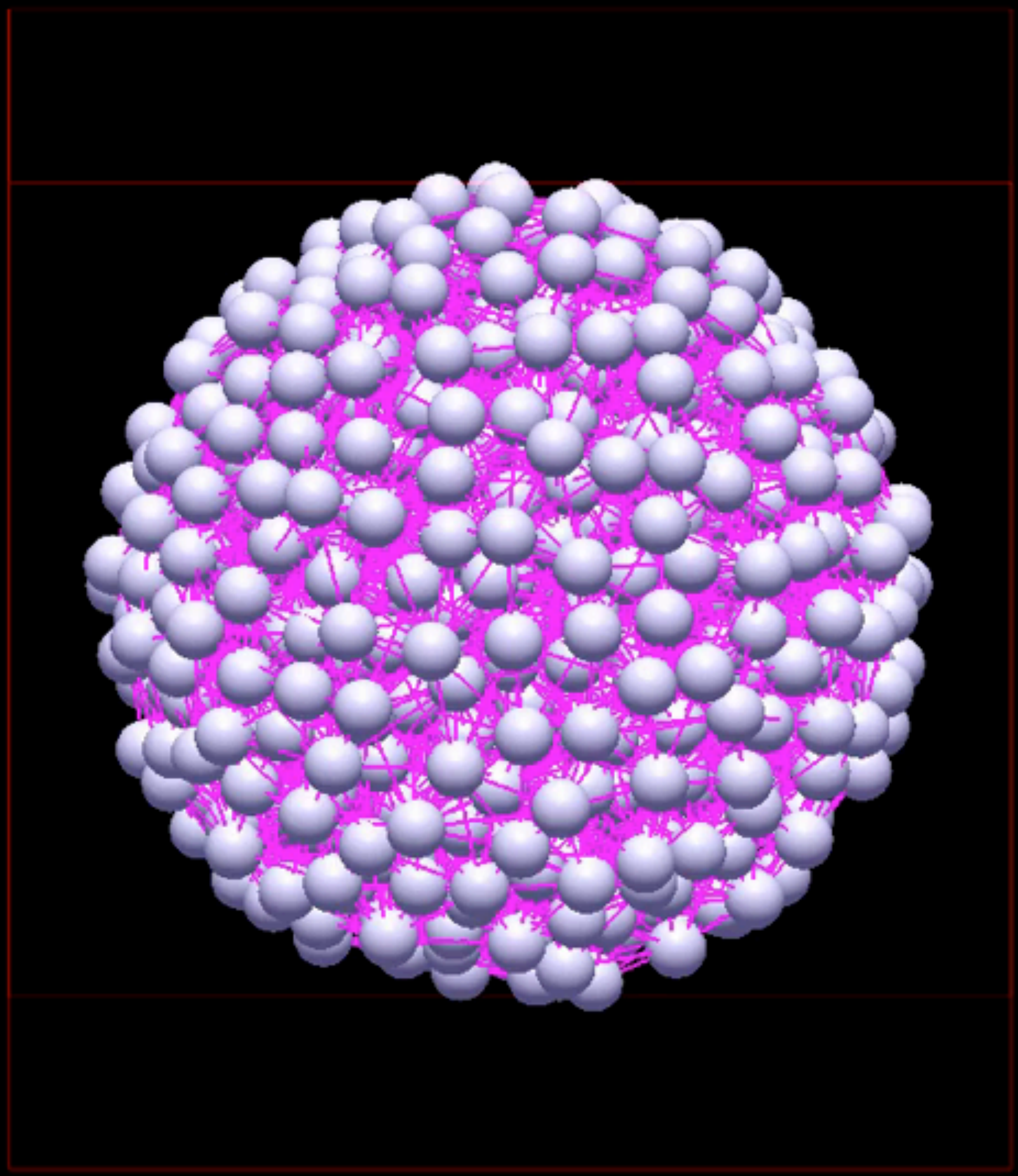
interior is *ductile*

Tectonics
(millions of years)



We need to simulate **brittle**/*elastic* phenomena
(and gravity)
–> NBody + springs (that could fail)

Mass Spring Model



Tidal encounters last less than 1 hour
High strain rate!
Elastic-Brittle regime (not plastic or ductile)

Mass-Spring model:
Gravity N-body
Rebound
+ inter-particle spring forces added

Young's modulus related to number density, lengths and strengths of springs

Damped mass-spring model within an N-body simulation

Spring force on body i from spring between i,j

$$\mathbf{F}^{elastic} = -k(L - L_{rest})\hat{\mathbf{n}}$$

- k: spring constant
- L: current length
- L_{rest}: rest spring length
- $\hat{\mathbf{n}}$: unit vector between bodies

Damping force law

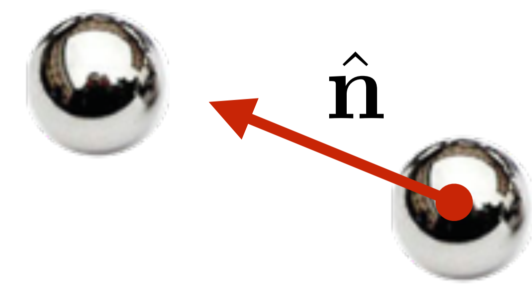
$$\mathbf{F}^{damping} = -\gamma\dot{\epsilon}L_{rest}m\hat{\mathbf{n}}$$

- γ: damping timescale
- $\dot{\epsilon}$: strain rate

$$\dot{\epsilon} = \frac{\dot{L}}{L_{rest}}$$

Damped mass-spring model within an N-body simulation

$$\mathbf{F}^{elastic} = -k(L - L_{rest})\hat{\mathbf{n}}$$



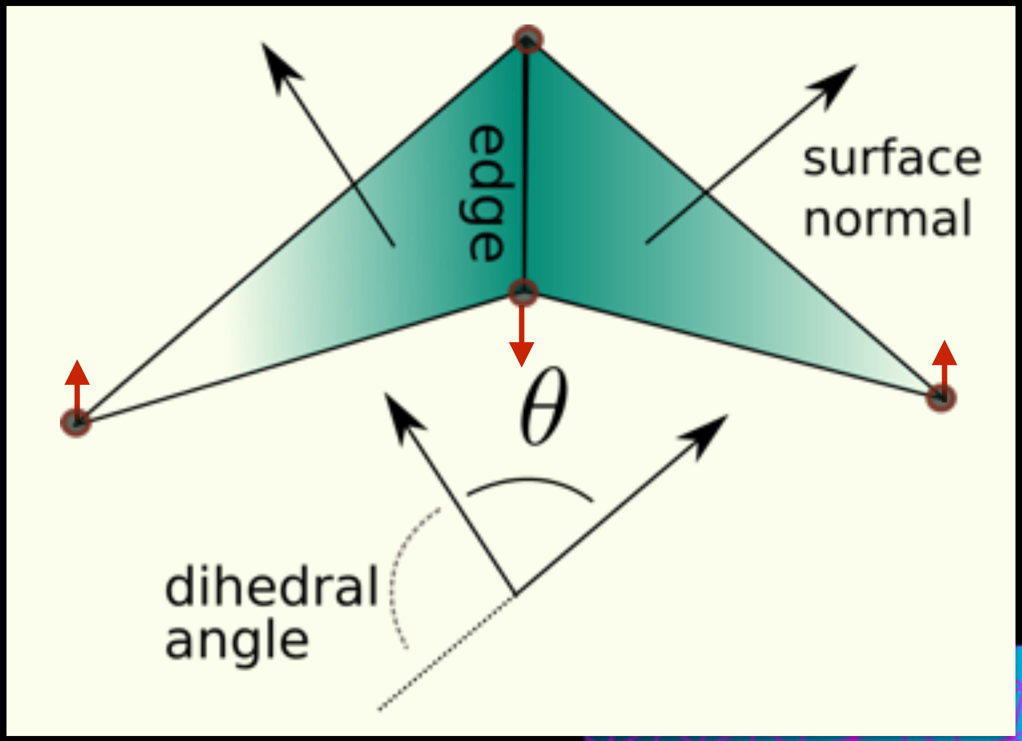
$$\mathbf{F}^{damping} = -\gamma\dot{\epsilon}L_{rest}m\hat{\mathbf{n}}$$

Forces on a pair of mass nodes are equal and opposite and in direction connecting the two nodes
--> Momentum and Angular momentum conservation assured.

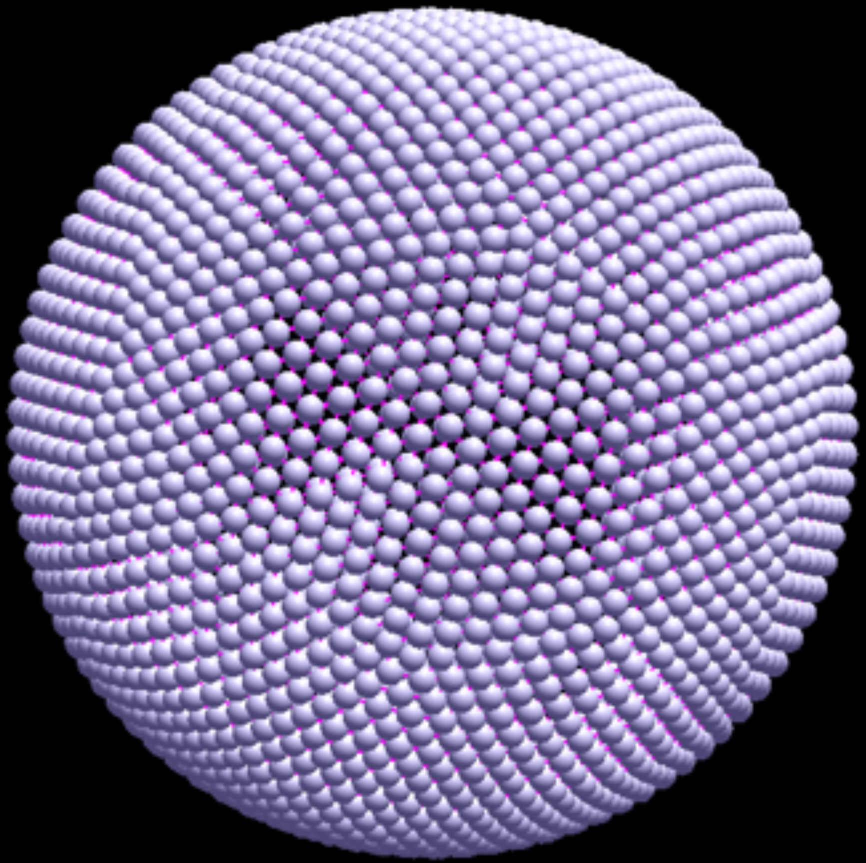
Distance between two mass nodes can be measured very **accurately**.

Nbody+springs: Why not used in astrophysics?

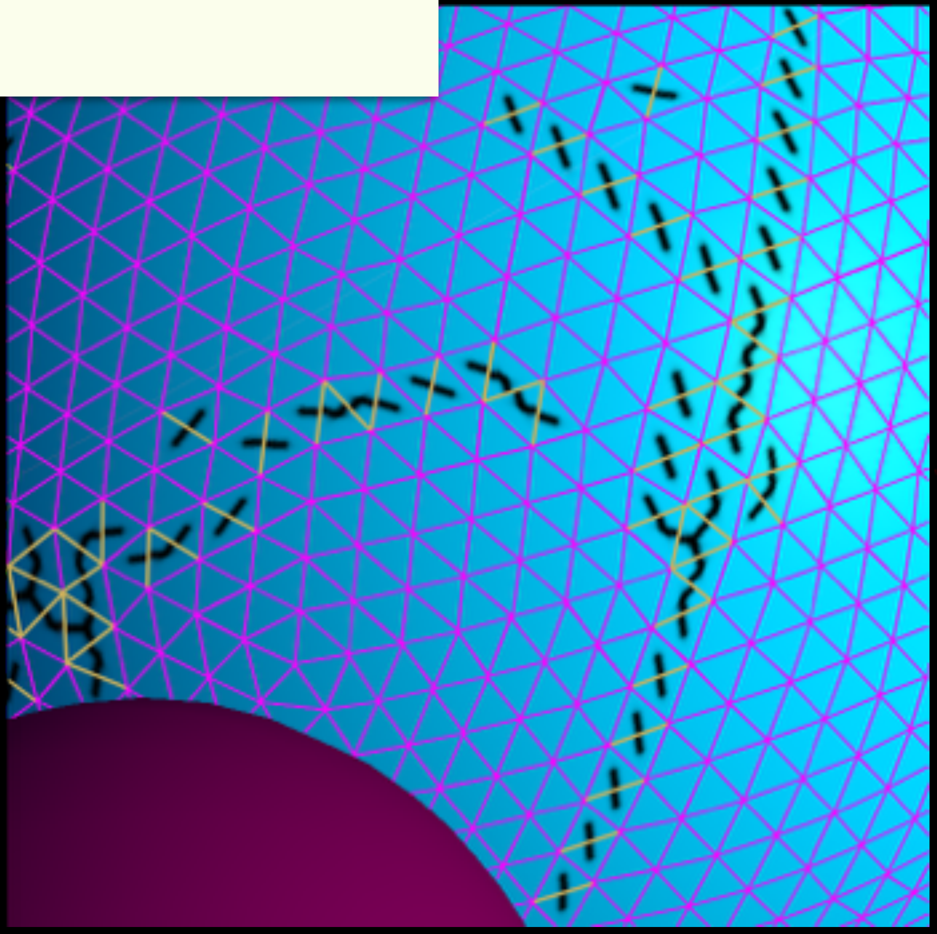
Crustal Model

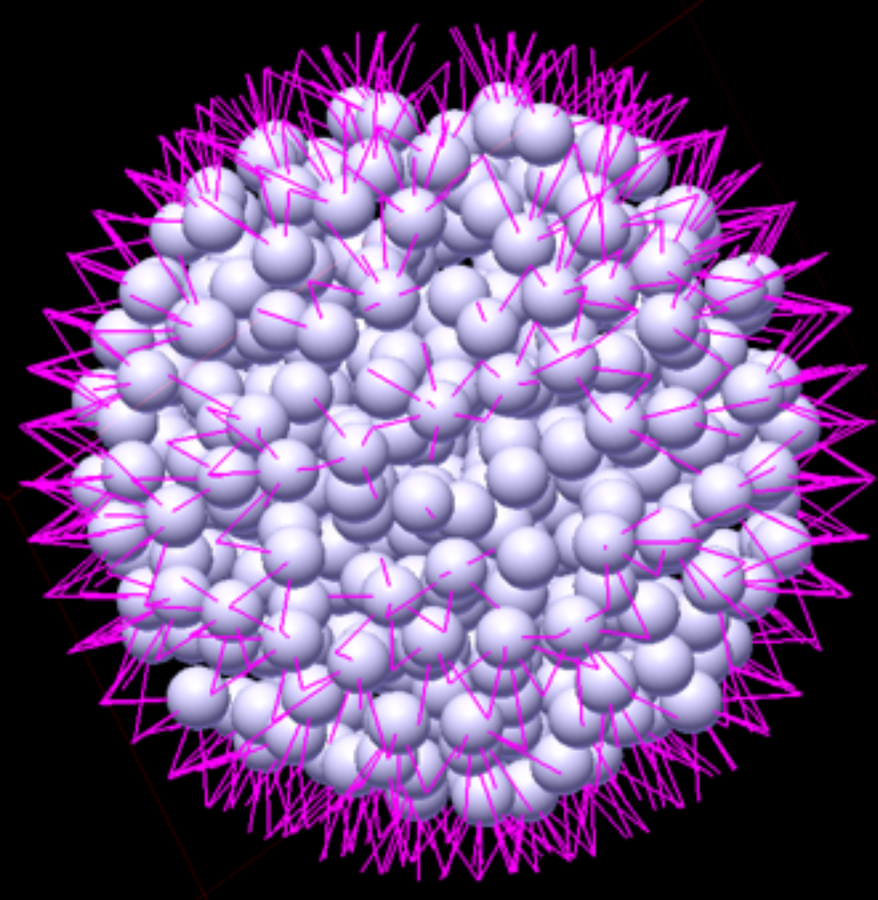


inspired by cloth modeling

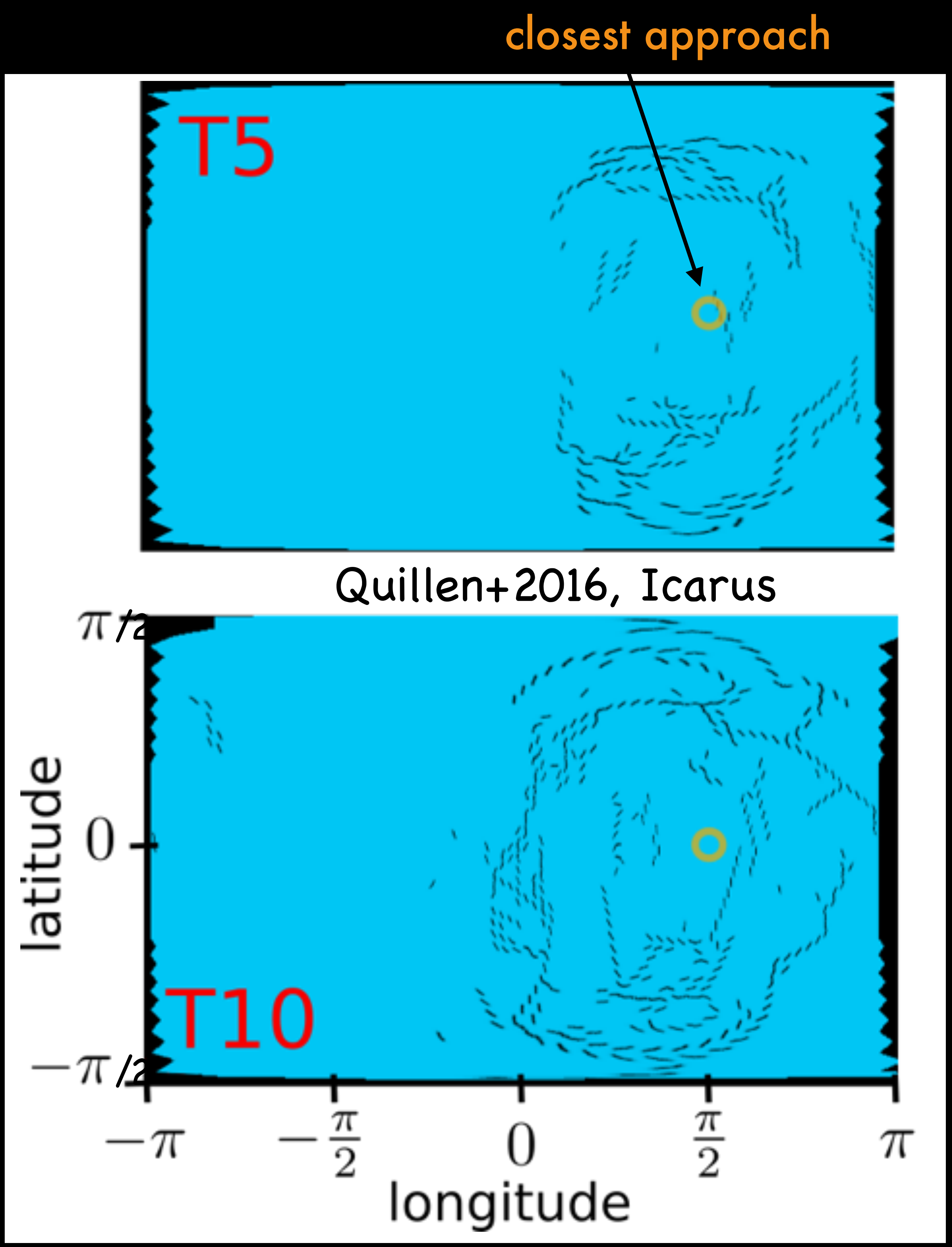


Spring strain maximum tensile failure criterion





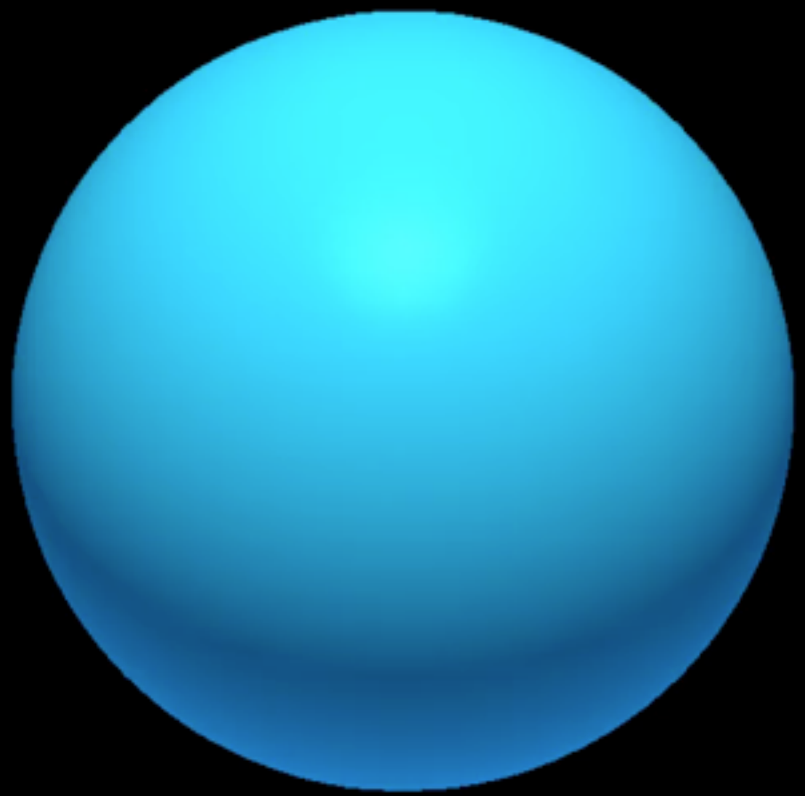
A very soft body



Harder body

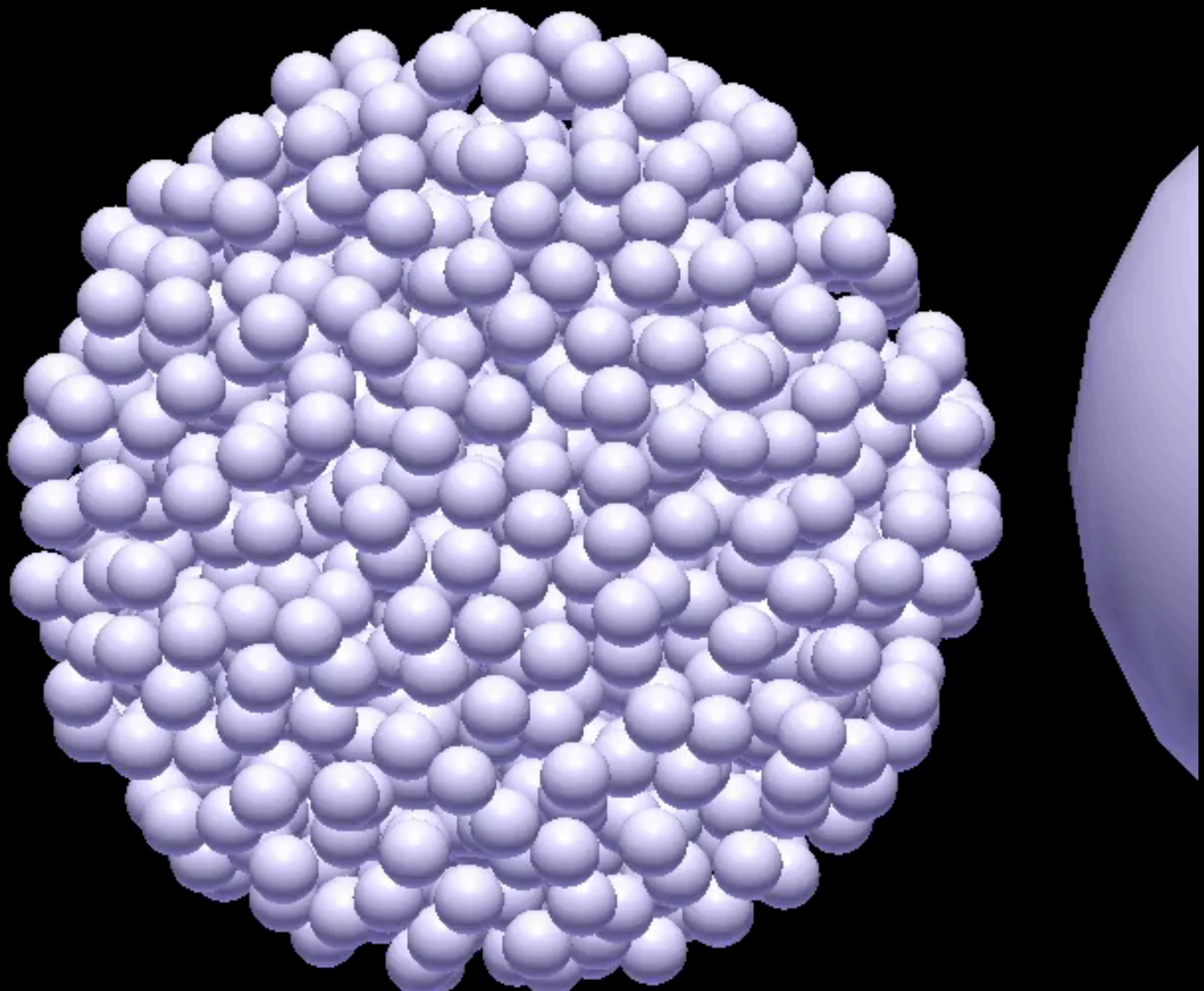
Regime Dione

$M_2 = 0.5M_1$

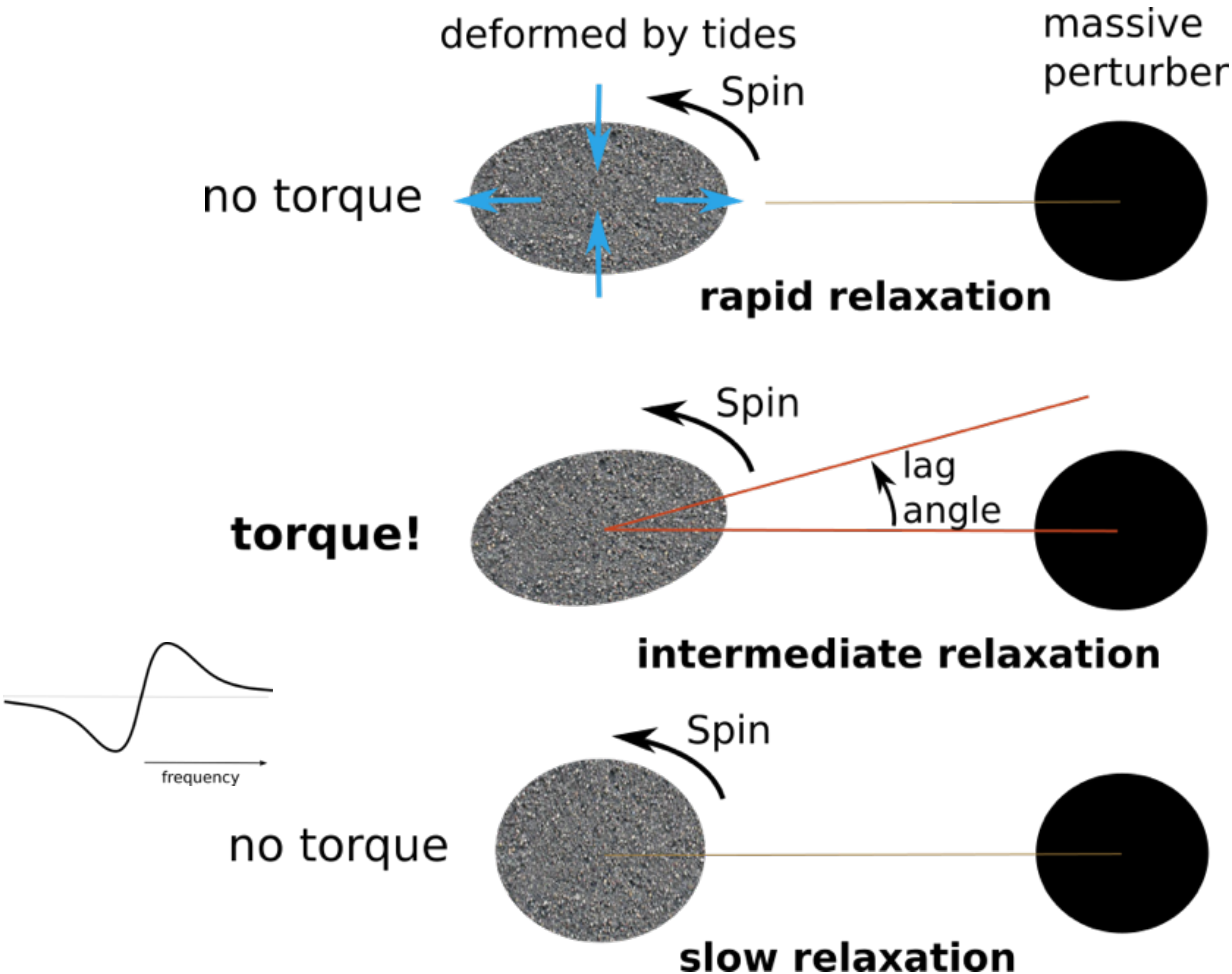


Summary — on tidal encounters

- Tidal encounters are a geophysical planetary process on large icy moons. A possible explanation for long grabens on **old** surfaces.
- Crustal tensile brittle failure can be caused by strong tidal encounters during the **early** solar system.
- Chasma extent: Larger than radius of body.
- Pattern of cracks: 1 hemisphere, large ring concentric around point of closest approach
- Resulting tectonic morphology of surface features (depth and width of chasms and grabens) yet to be predicted/modeled
- Possible application to Mars' Valles Marineris



movie of spin up — toward tidal lock


deformed by tides
massive perturber
Spin
no torque
rapid relaxation
Spin
lag angle
torque!
intermediate relaxation
frequency
Spin
no torque
slow relaxation

Tidal Torque

mass of perturber

$$\text{Torque} = \frac{3}{2}\frac{GM^{*2}}{a}\left(\frac{R}{a}\right)^6 [k \sin \epsilon](\omega)$$

semi-major axis

frequency of deformation in body frame (set by orbit and spin)

quality function

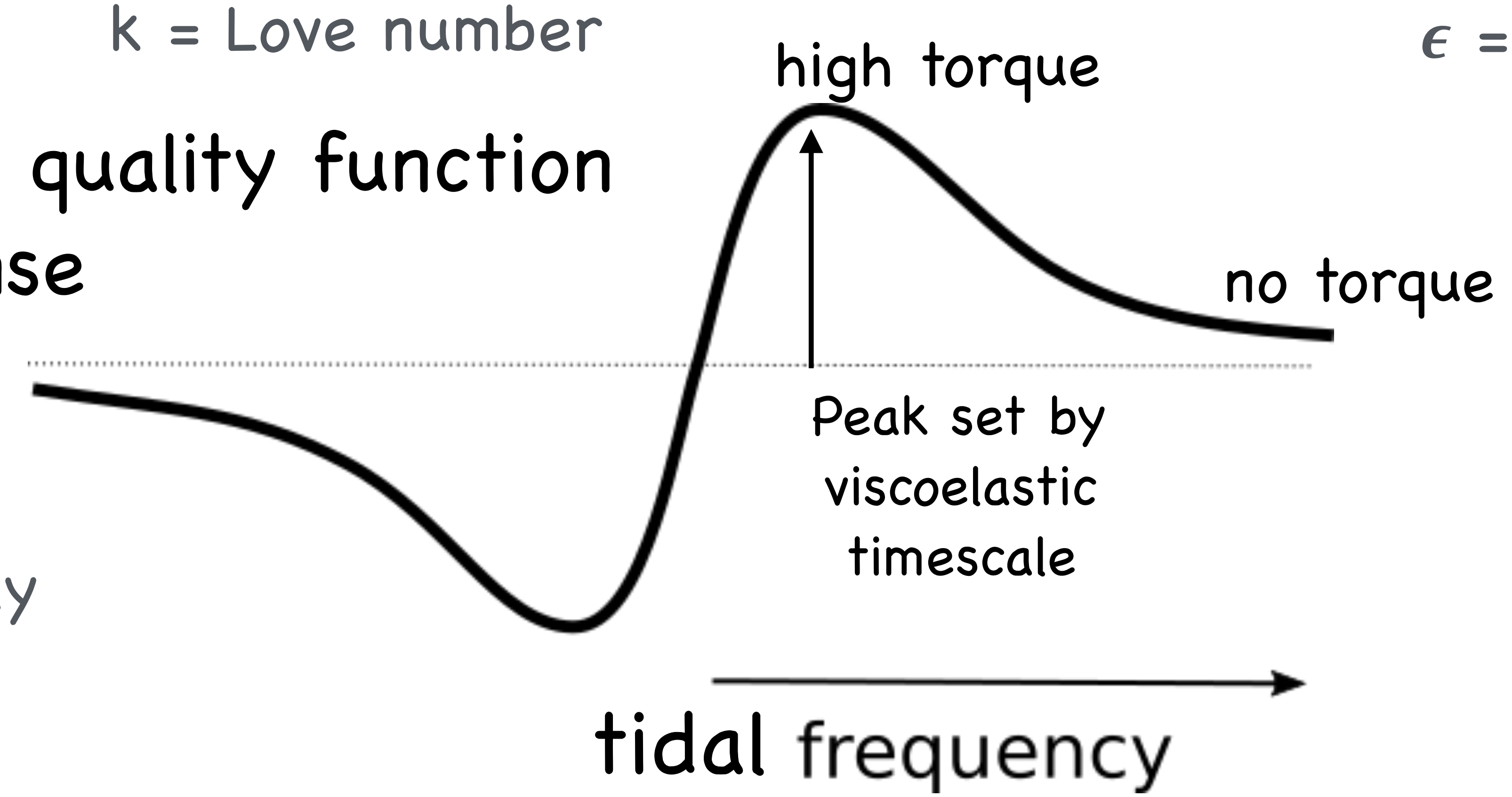
= product of deformation (k) and phase lag (ϵ)

k = Love number

$\epsilon = Q^{-1}$

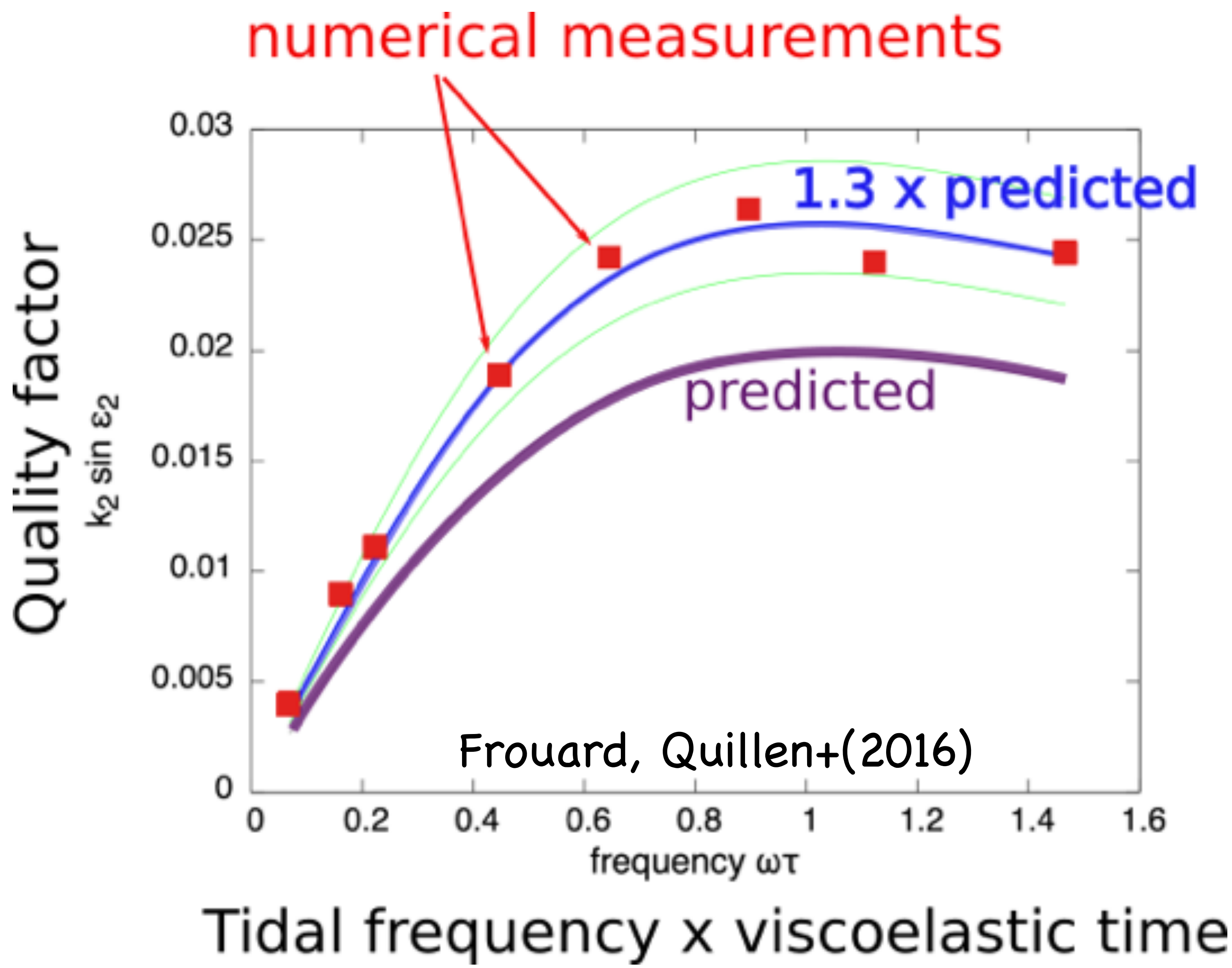
Expected shape of quality function

Viscoelastic response



See works by Efroimsky

Comparison of predicted vs numerically measured quality functions



Numerical measurements are too high by a factor of 1.3

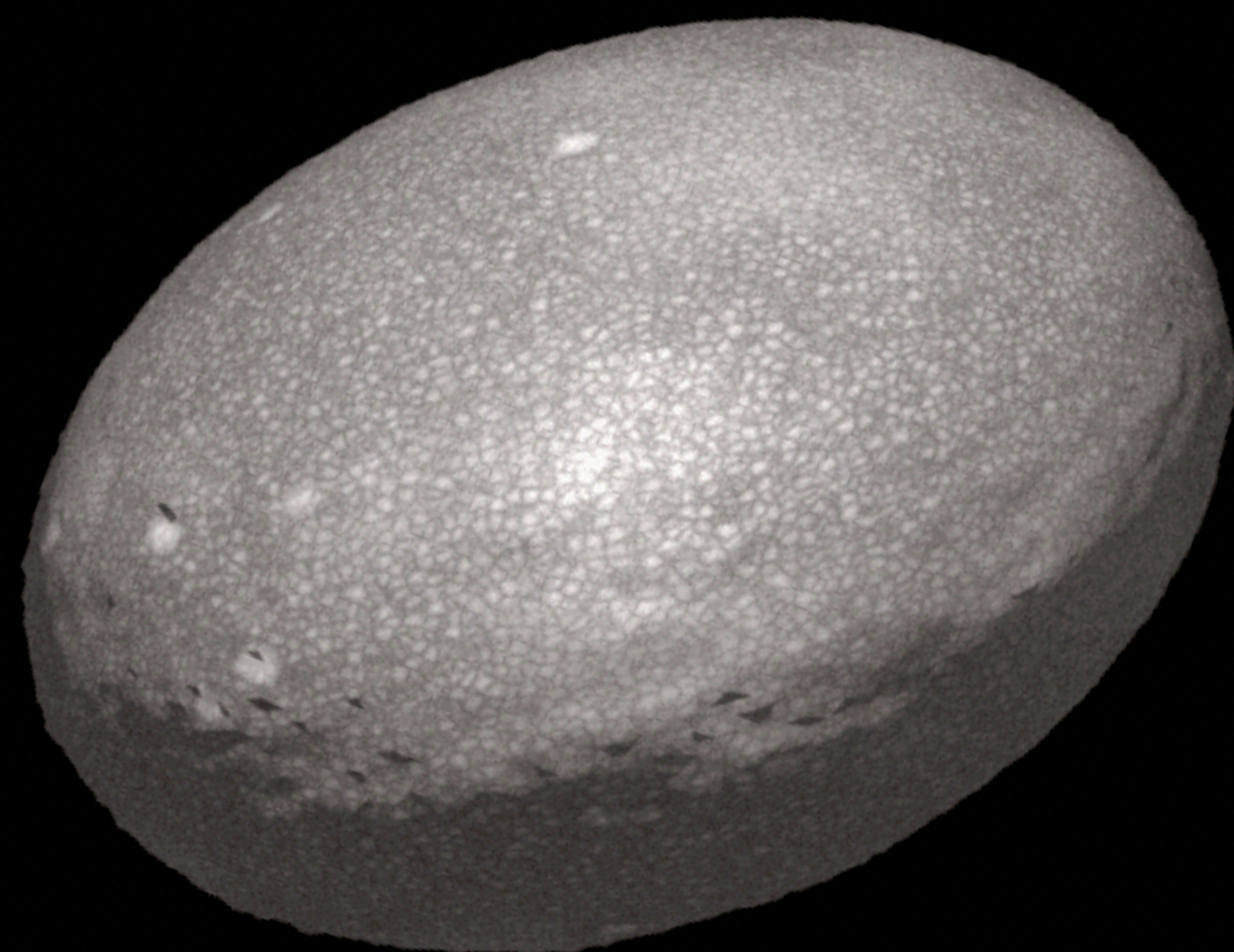
Peak is right place and viscoelastic response profile is seen! (we have estimated shear viscosity correctly)

Summary

- Tidal spin down simulated using simulated viscoelastic rheology. This could have been done in '70s.
- Material properties directly related to tidal response
- Predicted shape of viscoelastic response as a function of frequency is seen but with a 30% discrepancy
- We suspect that tidal analytical computations can be improved by including compressibility and associated bulk viscosity.

Haumea

4 hour spin rotation period

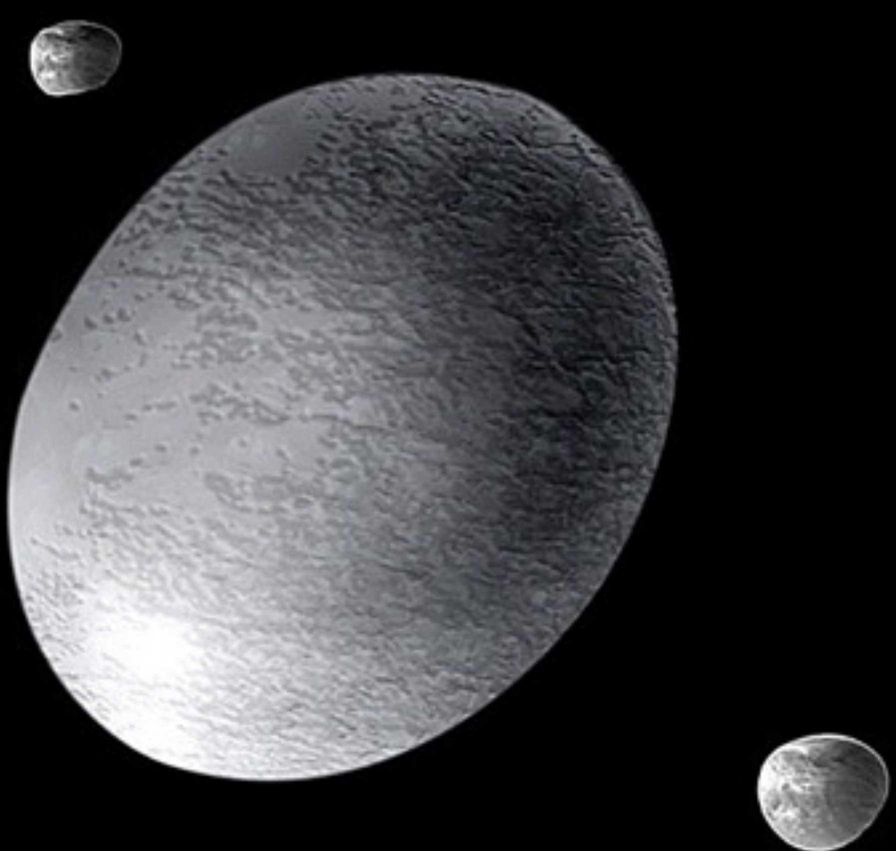


Axis ratios from light curve
b/a = 0.8   c/a = 0.5

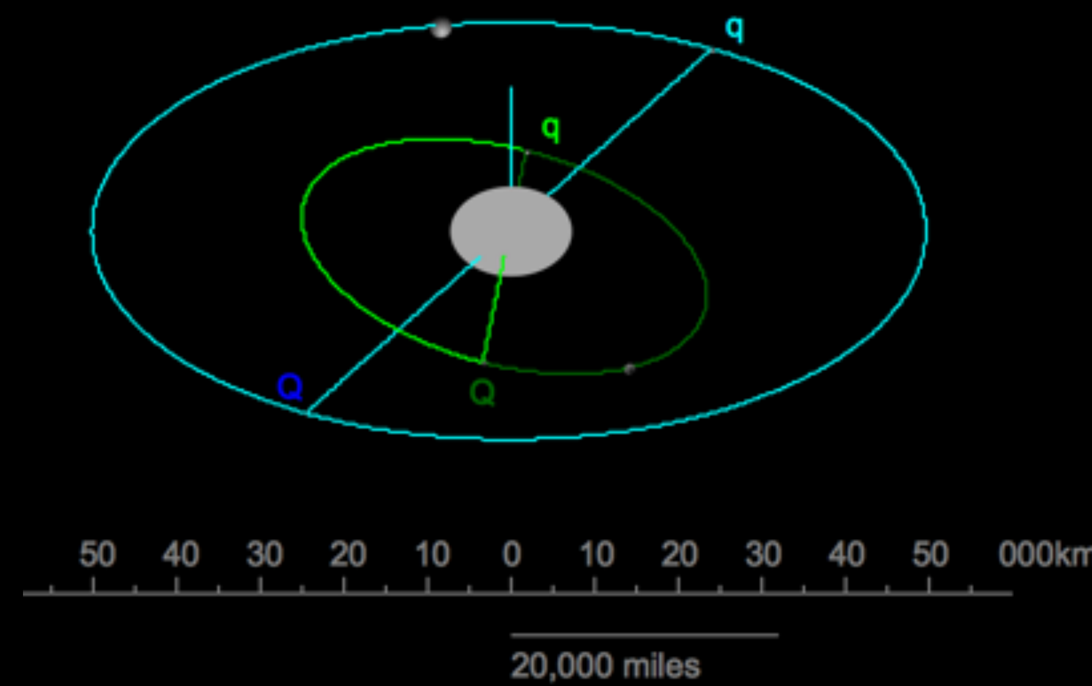
Movie by Stephanie Hoover

Haumea's two small moons: Hi'iaka and Namaka

mutual inclinations and
moderate eccentricities
Raggozine+09
Hi'iaka is outer satellite

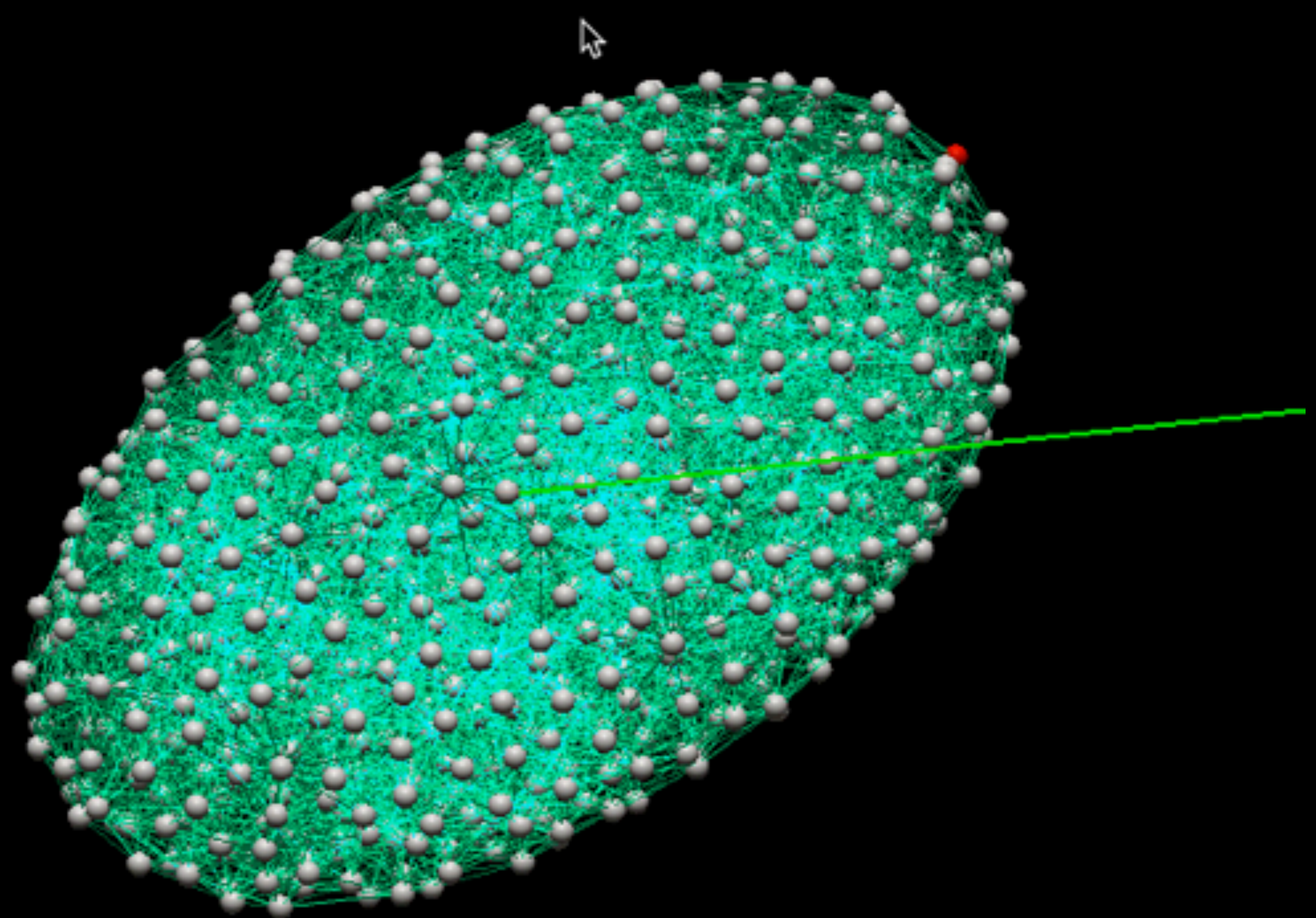


Artist conception



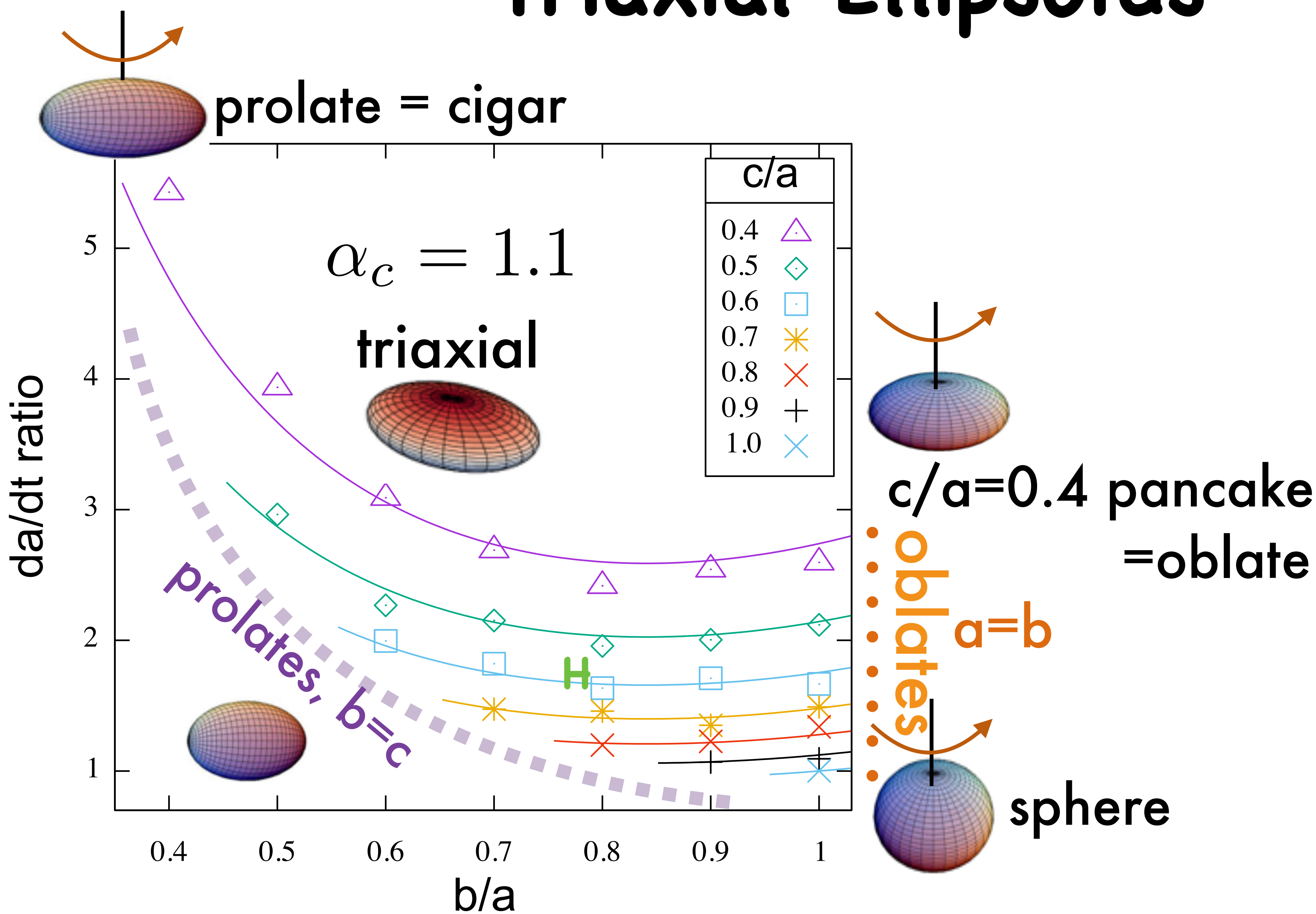
Born from a collision? (Leinhardt+)
Subsequent tidal evolution?
If Hi'iaka was born near Haumea
could it have tidally moved out to
current location?

Tidal spin down
of triaxial
ellipsoids
(Haumea)



Analytical predictions for torque
and spin down rate for ellipsoids are lacking

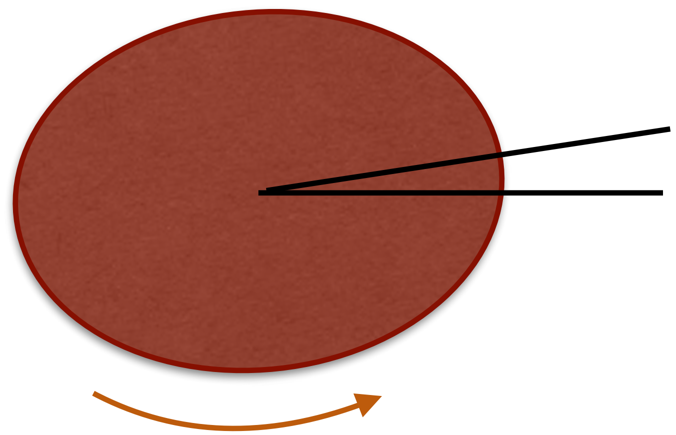
Triaxial Ellipsoids



$$\frac{\dot{a}_o}{\dot{a}_s} \approx \frac{1}{2}\left(1+\frac{b^4}{a^4}\right)\left(\frac{b}{a}\right)^{-\frac{4}{3}}\left(\frac{c}{a}\right)^{-\alpha_c}$$

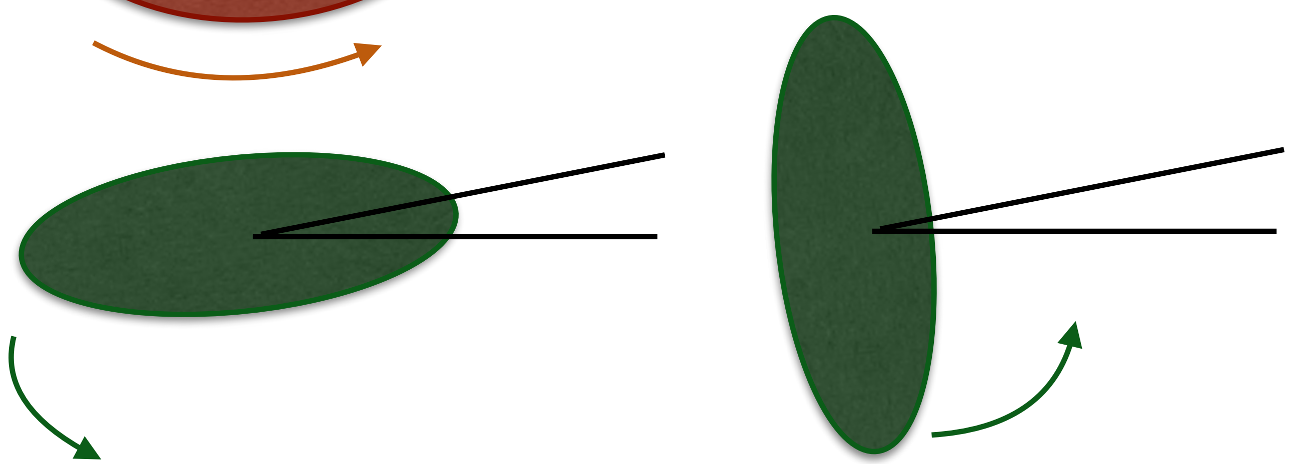
Quillen+2016b, MNRAS

Estimated Torques/orbital drift rates



Oblate body is made slightly triaxial by tidal force

Shape and tilt same at all times in orbit



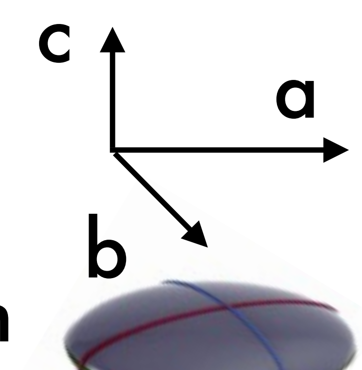
Prolate body:

average torque for two perpendicular orientations

$$\frac{\dot{a}_o}{\dot{a}_s} \approx \frac{1}{2}\left(1+\frac{b^4}{a^4}\right)\left(\frac{b}{a}\right)^{-\frac{4}{3}} \left(\frac{c}{a}\right)^{-\frac{4}{3}}$$

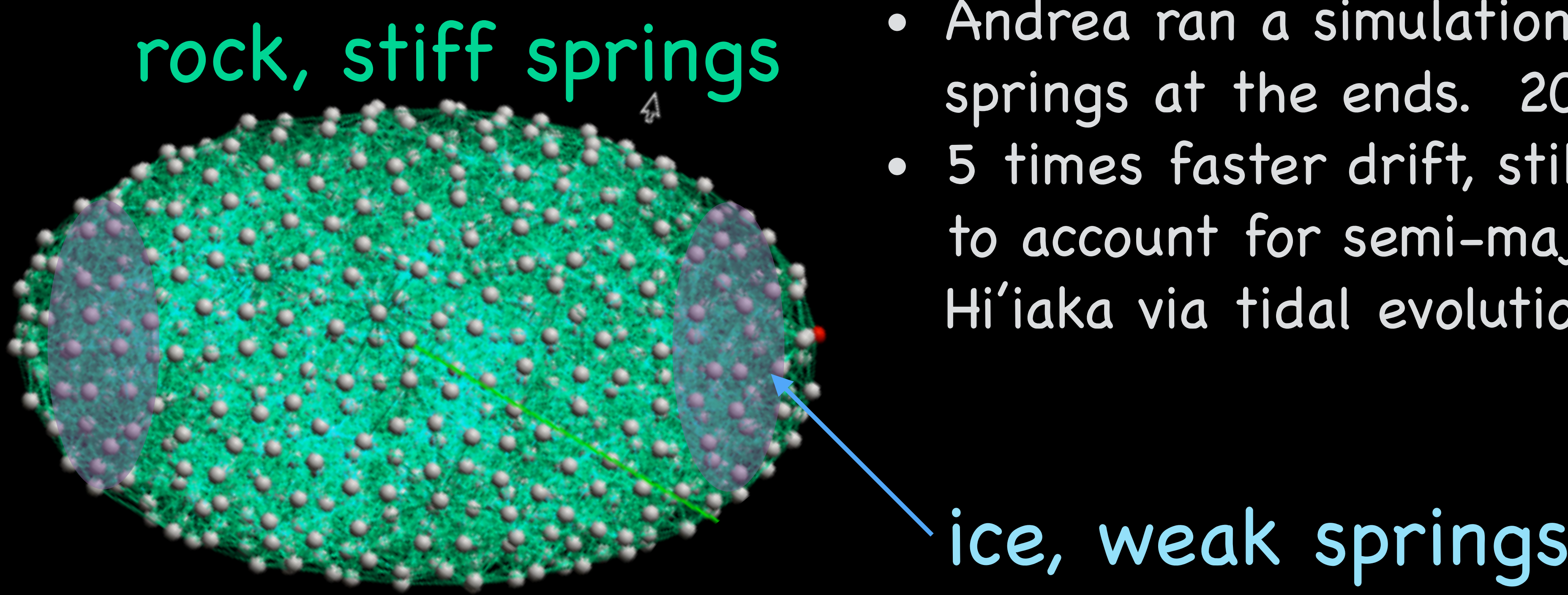
prolate average

cross section areas



Tidal Evolution of Haumea/Hi'iaka

- Homogeneous ellipsoid with same axis ratios as Haumea drifts twice as fast as equivalent volume sphere
- Cannot account for current semi-major axis of Hi'iaka, assuming born close together, and tidally drifted to current location
- Kondratyev (2016) proposed that stresses between icy shell and core and associated relaxation would cause ice to accumulate at the ends of Haumea.

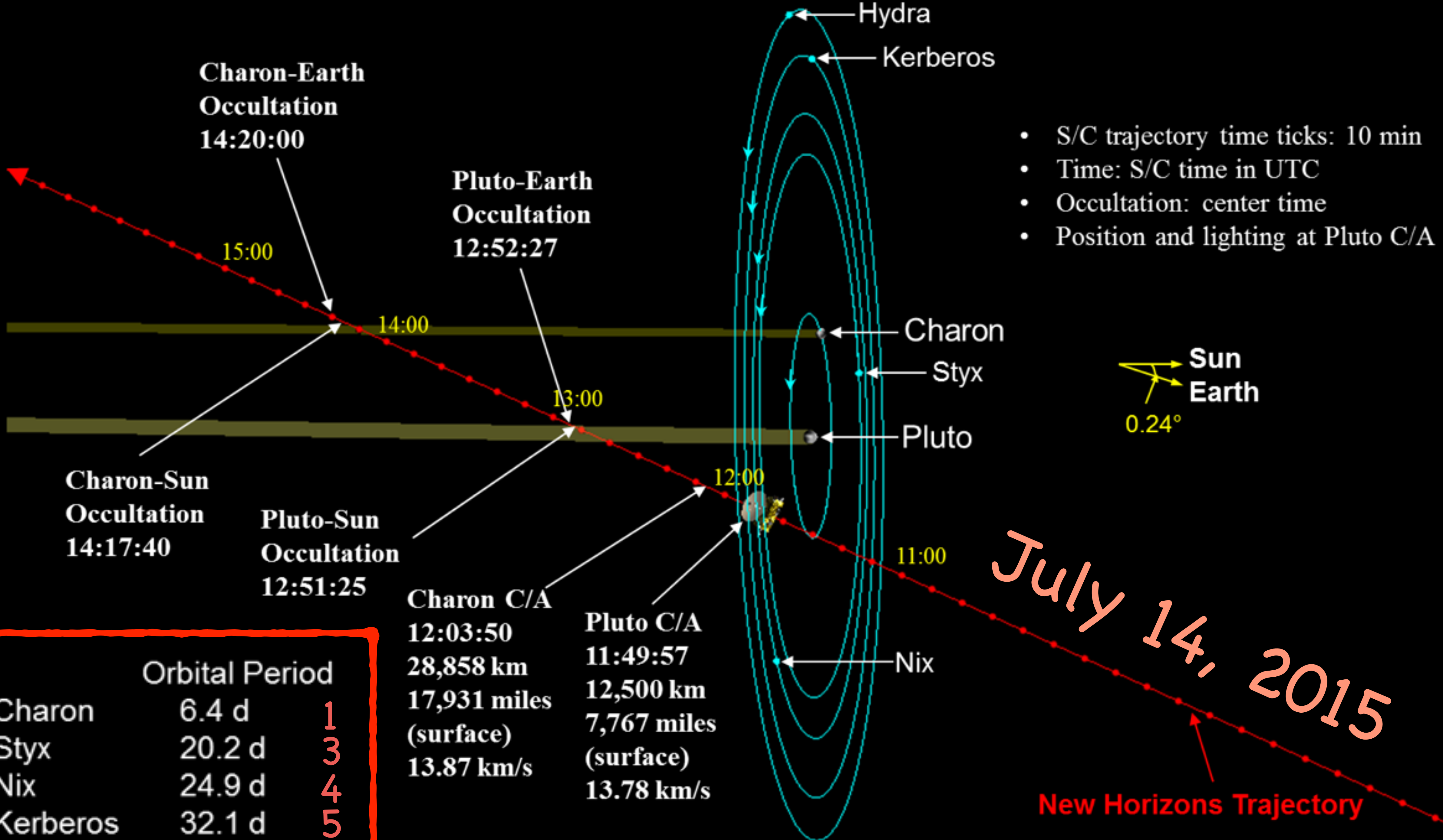


- Andrea ran a simulation with weaker springs at the ends. 20% ice.
- 5 times faster drift, still not enough to account for semi-major axis of Hi'iaka via tidal evolution alone

Summary

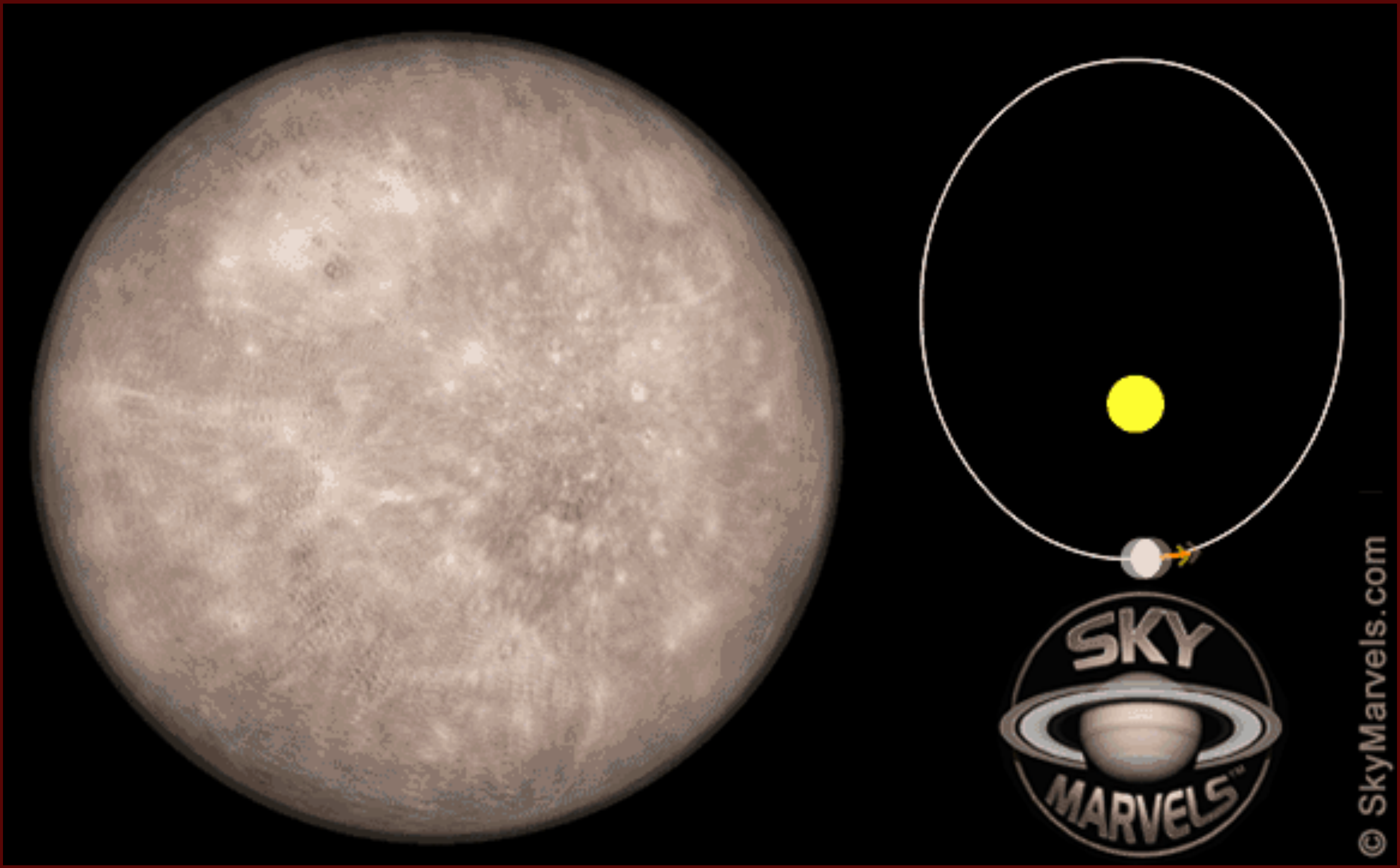
- We can simulate tidal evolution of inhomogeneous, non-round and elastically anisotropic bodies
- Scaling of measurements motivated us to understand approximate scaling through stress/strain relation (3D generalization of Hooke's law)

New Horizons Mission



	Orbital Period	
Charon	6.4 d	1
Styx	20.2 d	3
Nix	24.9 d	4
Kerberos	32.1 d	5
Hydra	38.2 d	6

Spin orbit resonance – Mercury



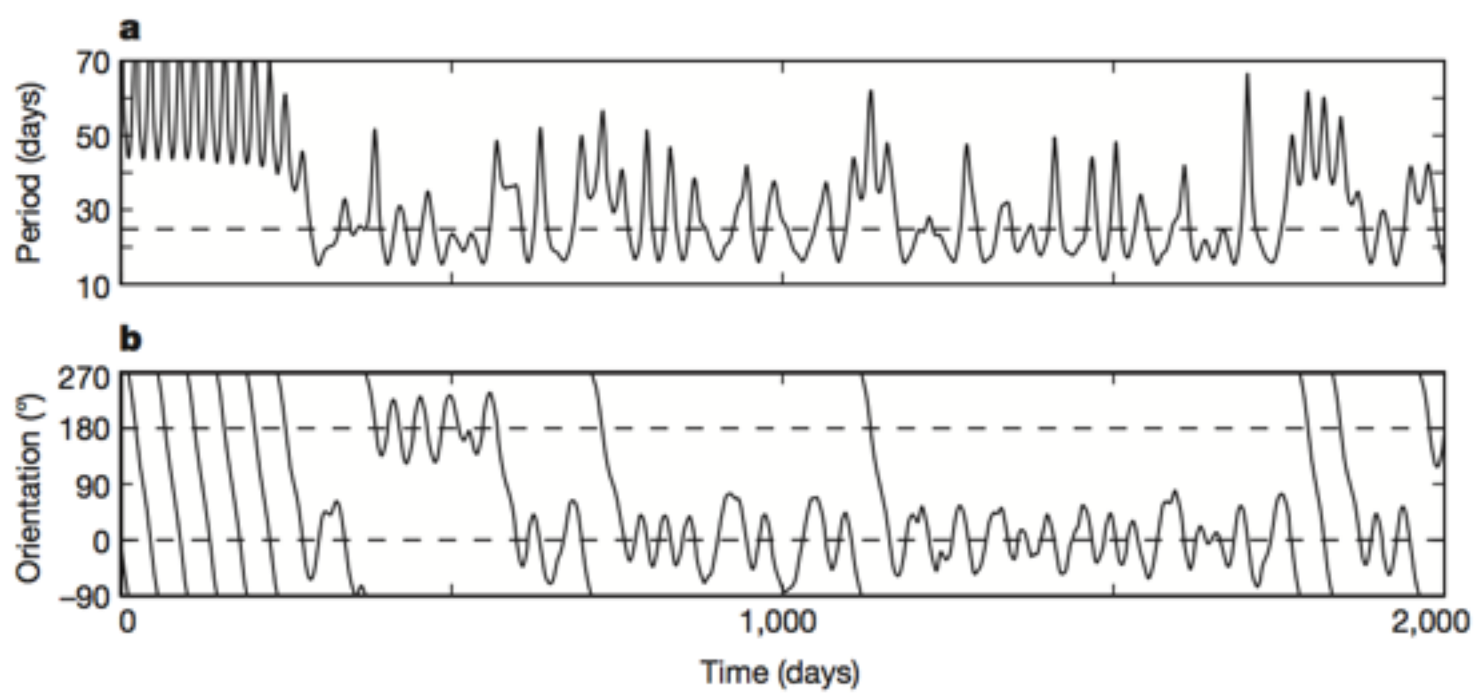
Predictions prior to arrival of New Horizons Mission at Pluto

doi:10.1038/nature14469

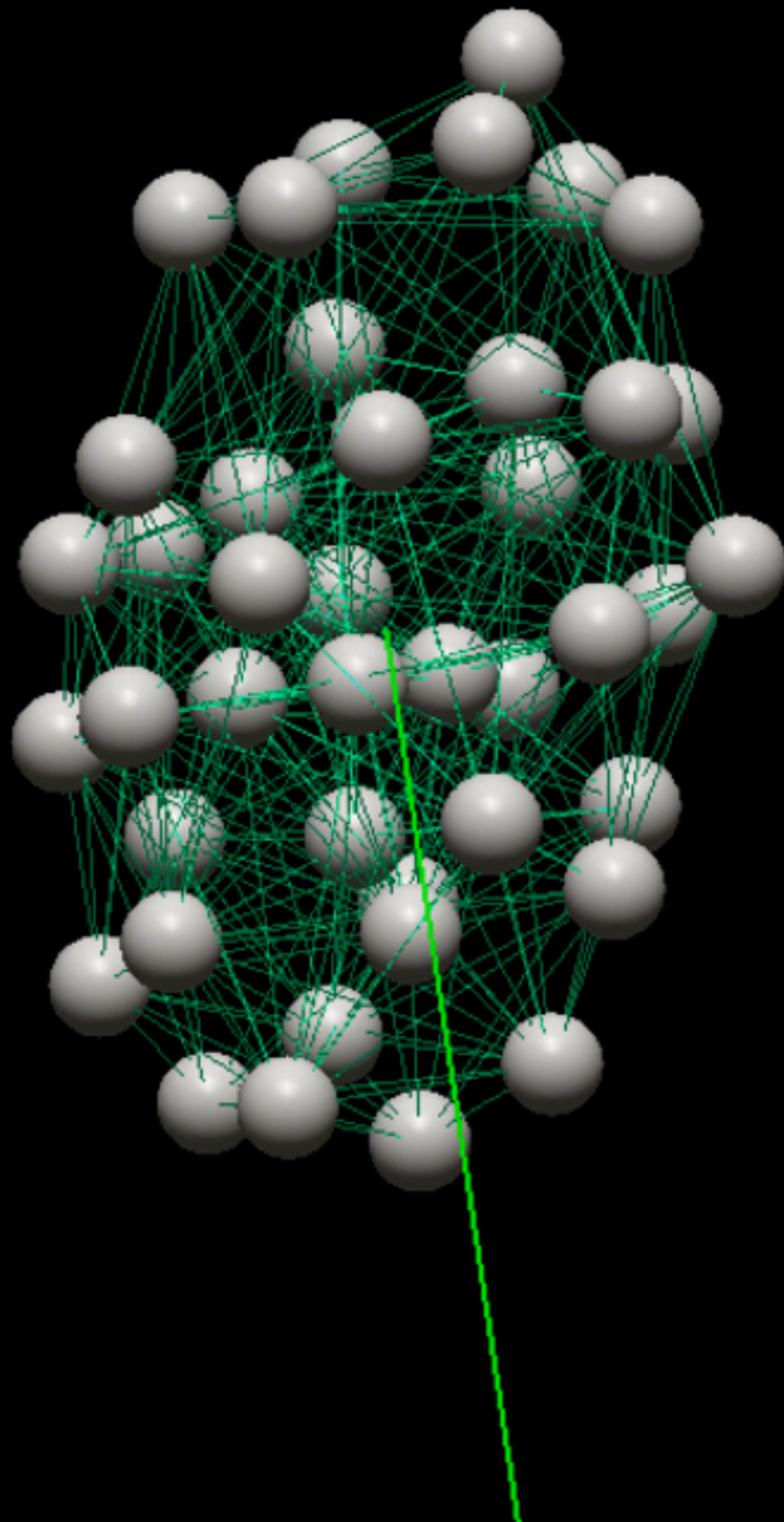
Resonant interactions and chaotic rotation of Pluto's small moons

M. R. Showalter[1] & D. P. Hamilton[2]

Four small moons—Styx, Nix, Kerberos and Hydra—follow near-circular, near-equatorial orbits around the central 'binary planet' comprising Pluto and its large moon, Charon. New observational details of the system have emerged following the discoveries of Kerberos and Styx. Here we report that Styx, Nix and Hydra are tied together by a three-body resonance, which is reminiscent of the Laplace resonance linking Jupiter's moons Io, Europa and Ganymede. Perturbations by the other bodies, however, inject chaos into this otherwise stable configuration. Nix and Hydra have bright surfaces similar to that of Charon. Kerberos may be much darker, raising questions about how a heterogeneous satellite system might have formed. Nix and Hydra rotate chaotically, driven by the large torques of the Pluto–Charon binary.



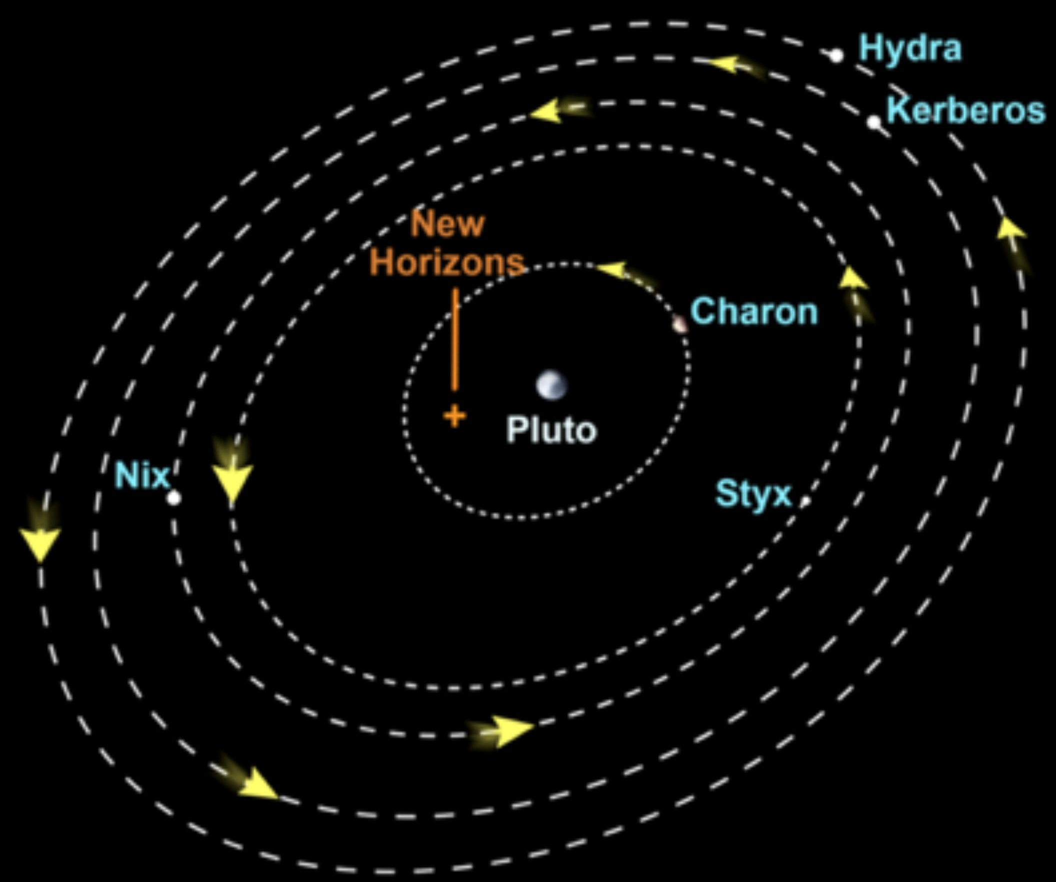
Tumbling

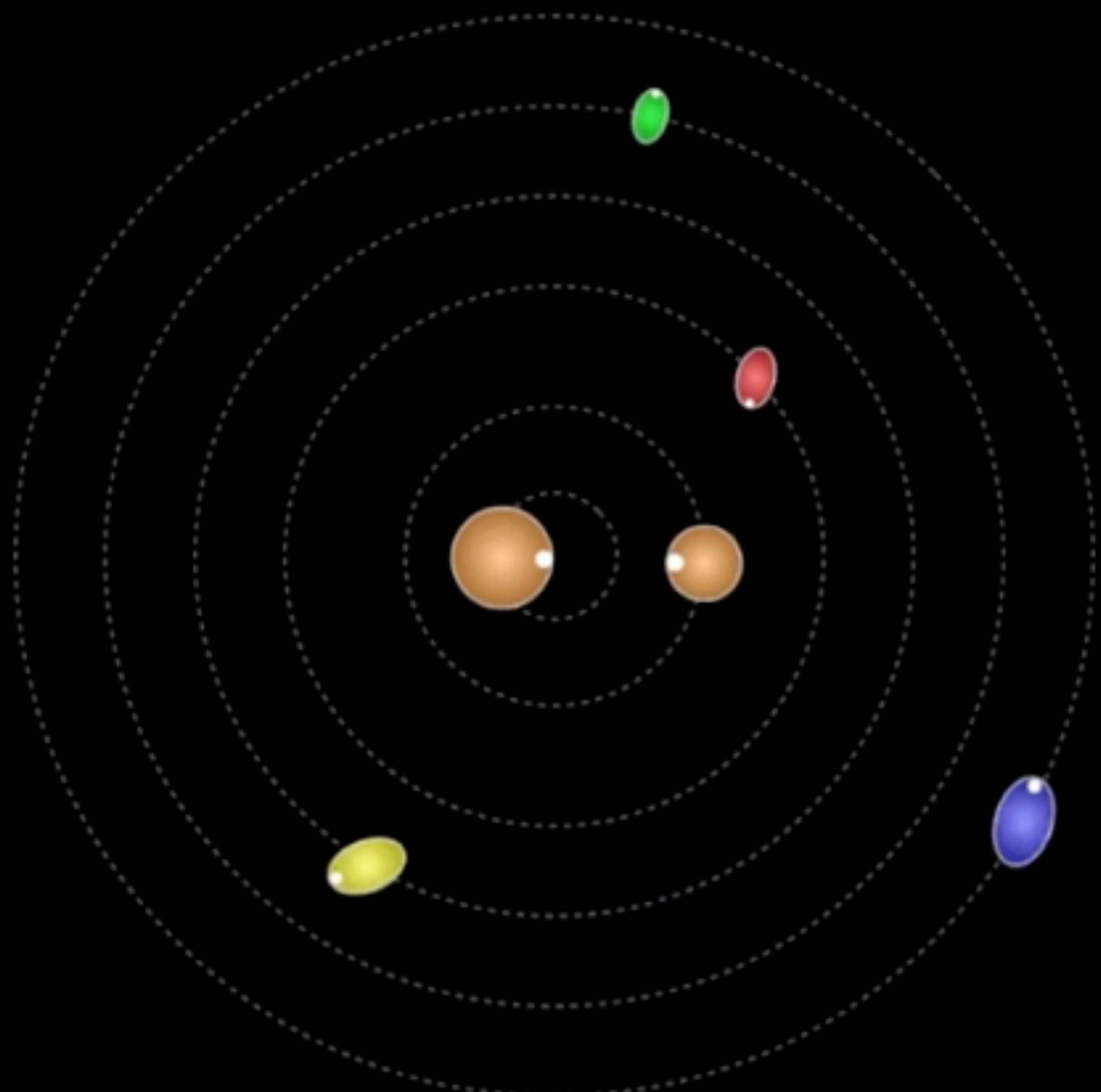


Tumbling predicted for Pluto's Minor Satellites by Showalter+2015 Correia+2015 (theory) because of binary perturbations and elongated body shapes.

Spin orbit resonances overlap causing chaotic tumbling of Hyperion (Jack Wisdom)
There is no tidal lock

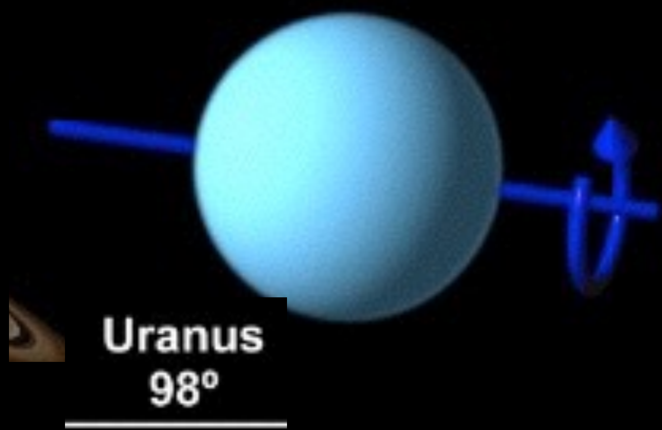
New Horizons visit to Pluto

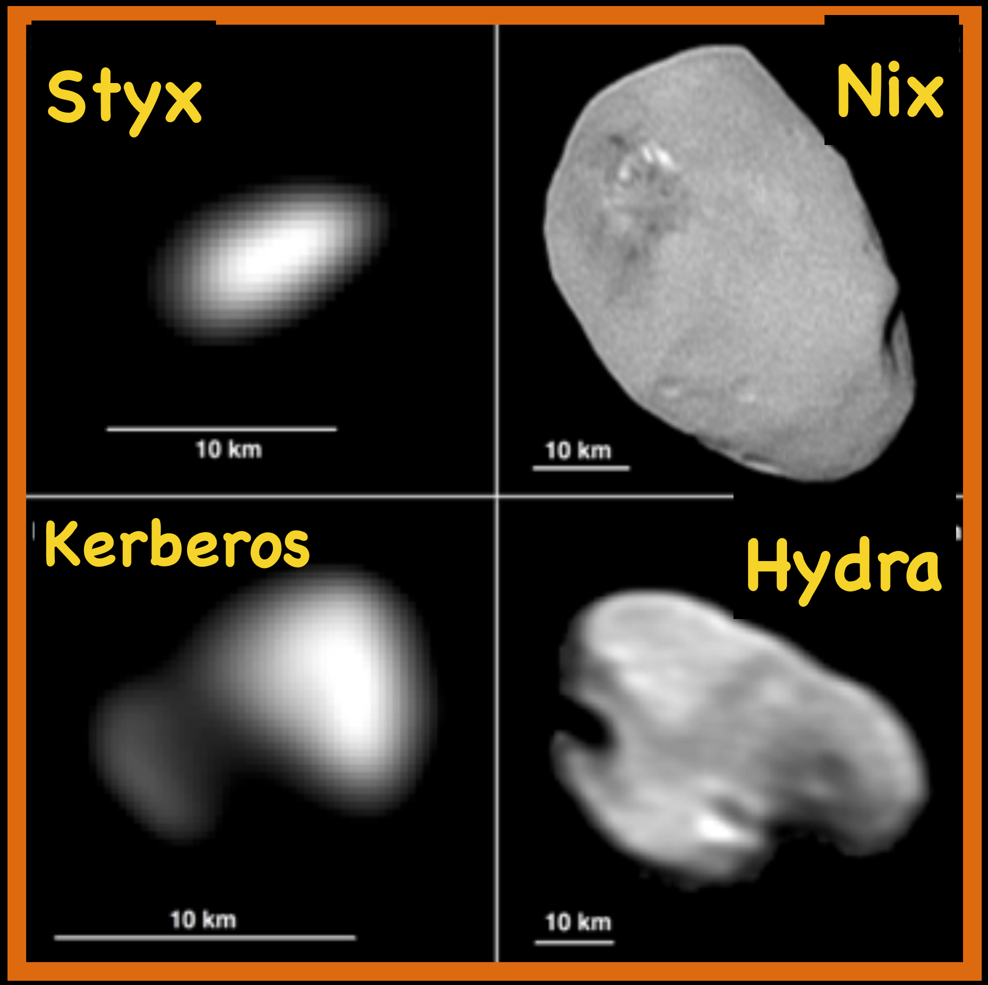


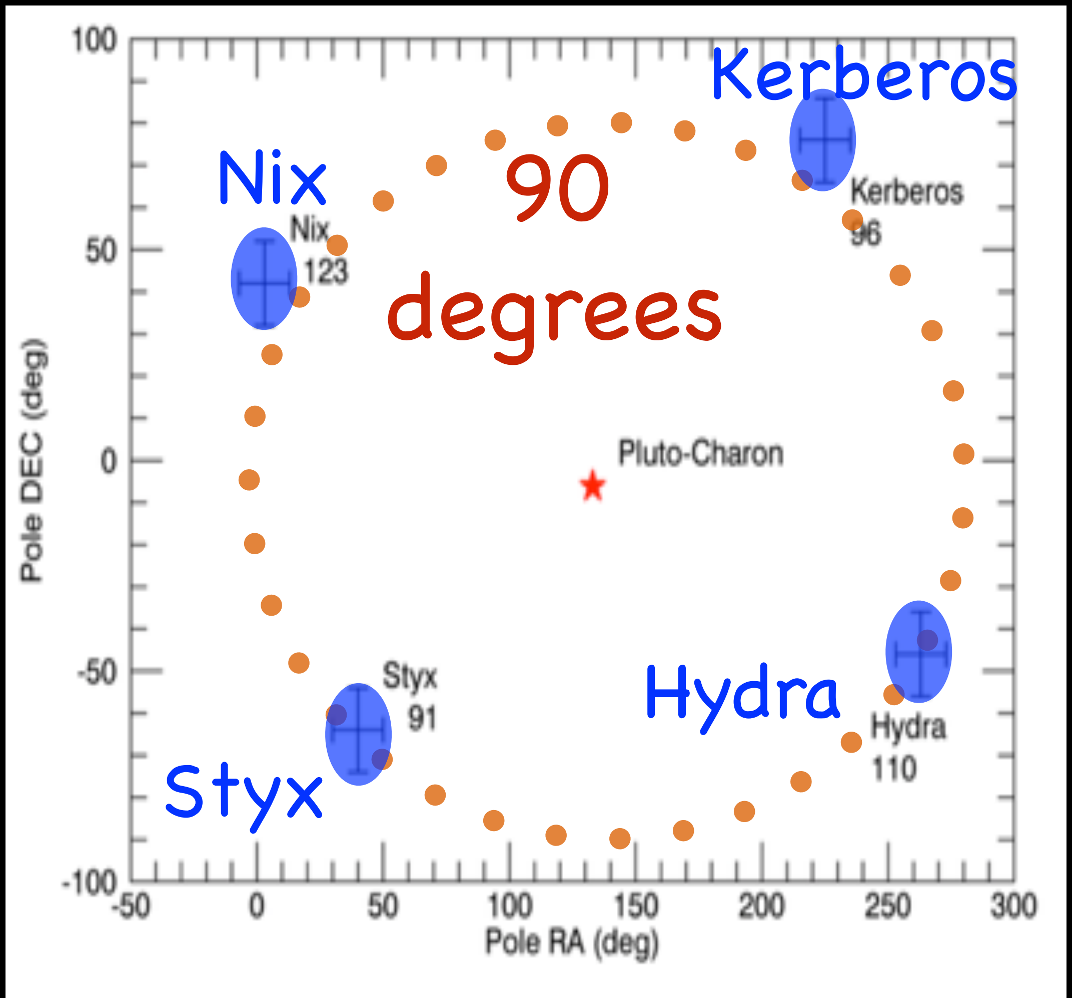


Mark Showwalter
Weaver+2016

Pluto+Charon's small satellites are like Uranus, 90-120° obliquities w.r.t to orbit







Weaver+2016

Styx, Nix, Kerberos, Hydra:
Are the Obliquities Primordial?

Born in a disk

likely because of low eccentricity orbits

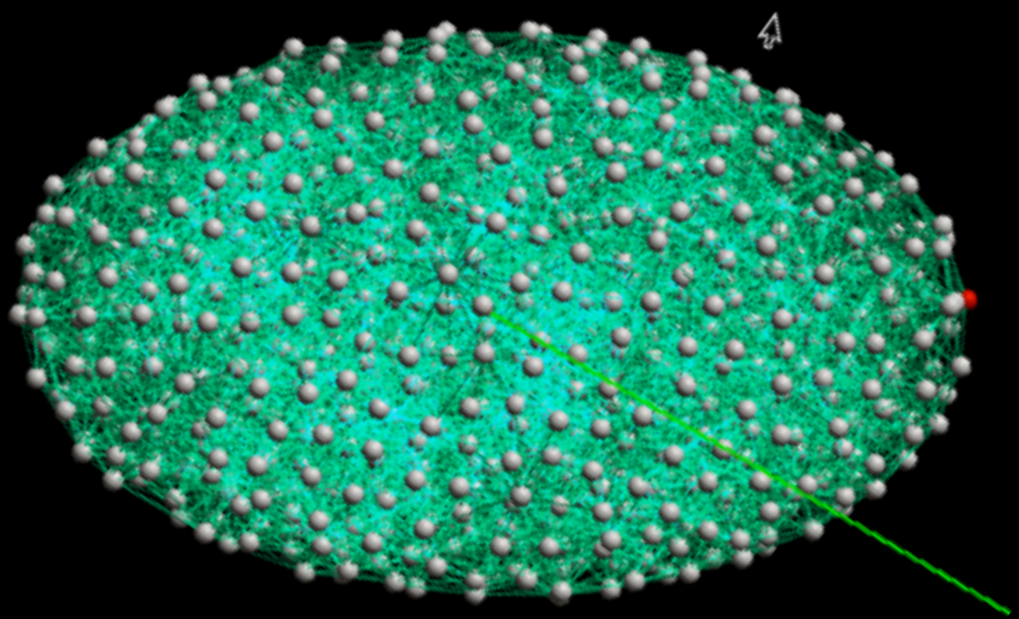
and near resonant chain orbits

Accretion from gas → low obliquities

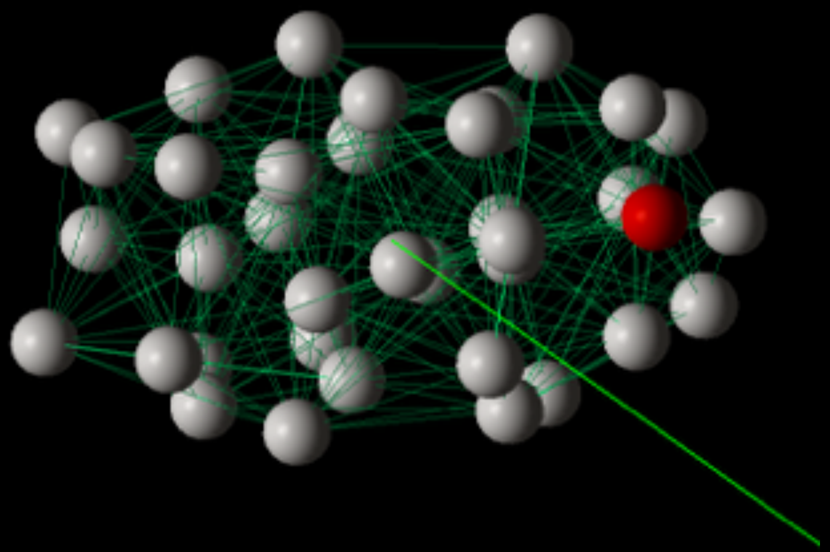
Late stage accretion from large planetesimals → random obliquities

Current obliquities are clustered near 90°

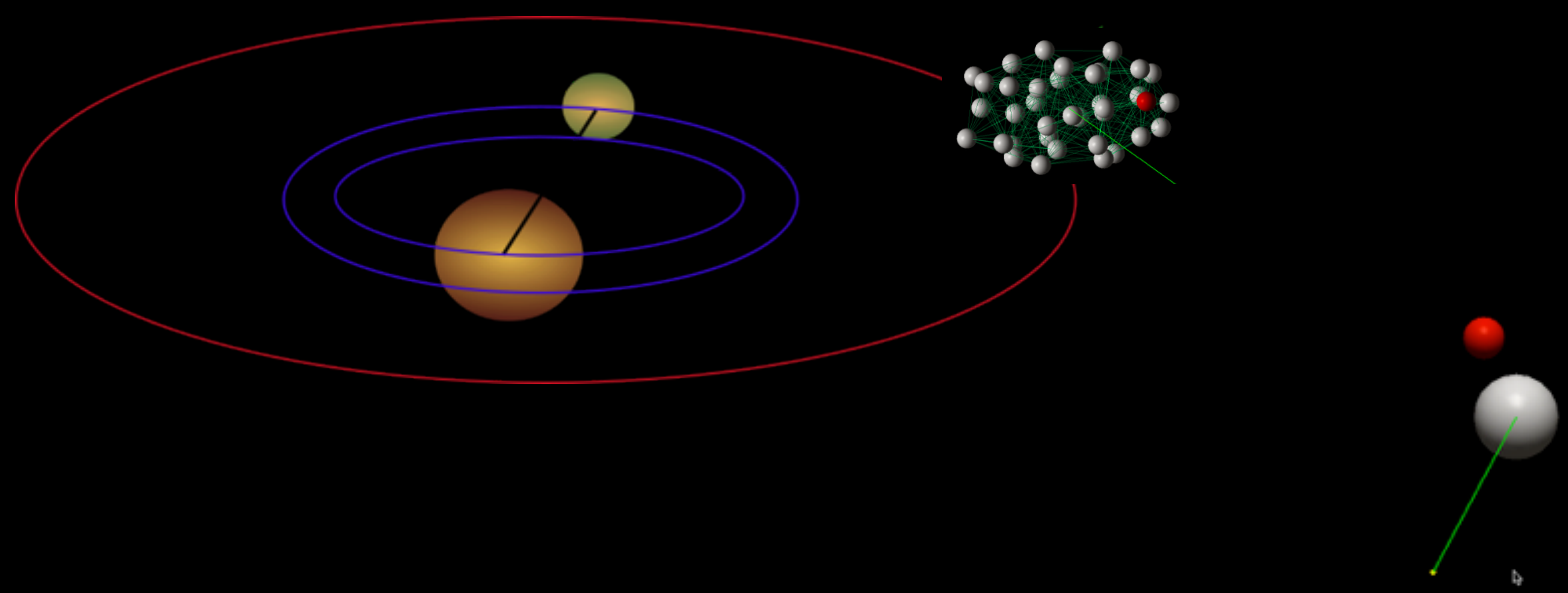
—> Explore mechanisms for obliquity variation simulate a few dozen orbits



simulate a 100,000 orbits

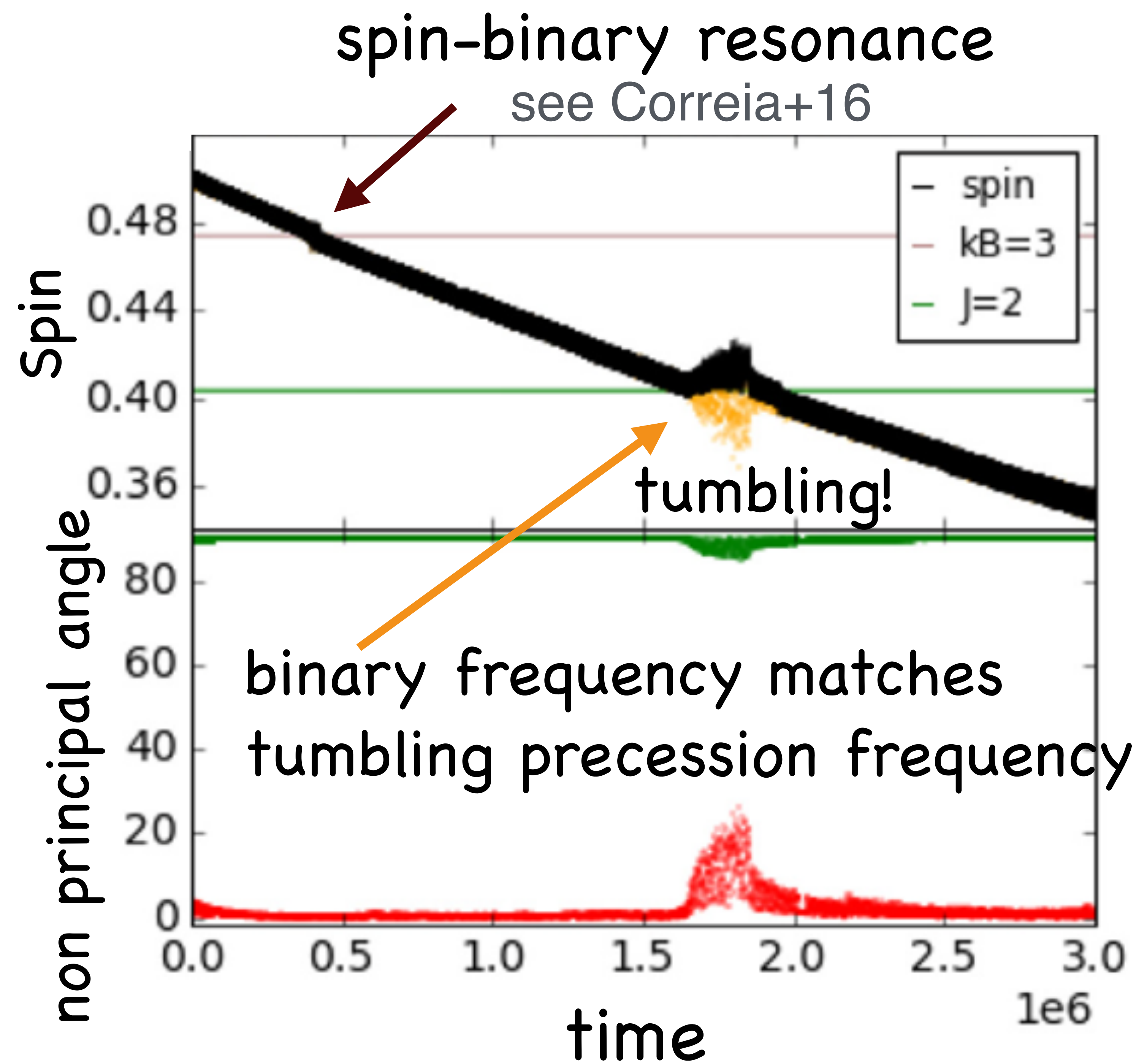


Fiona

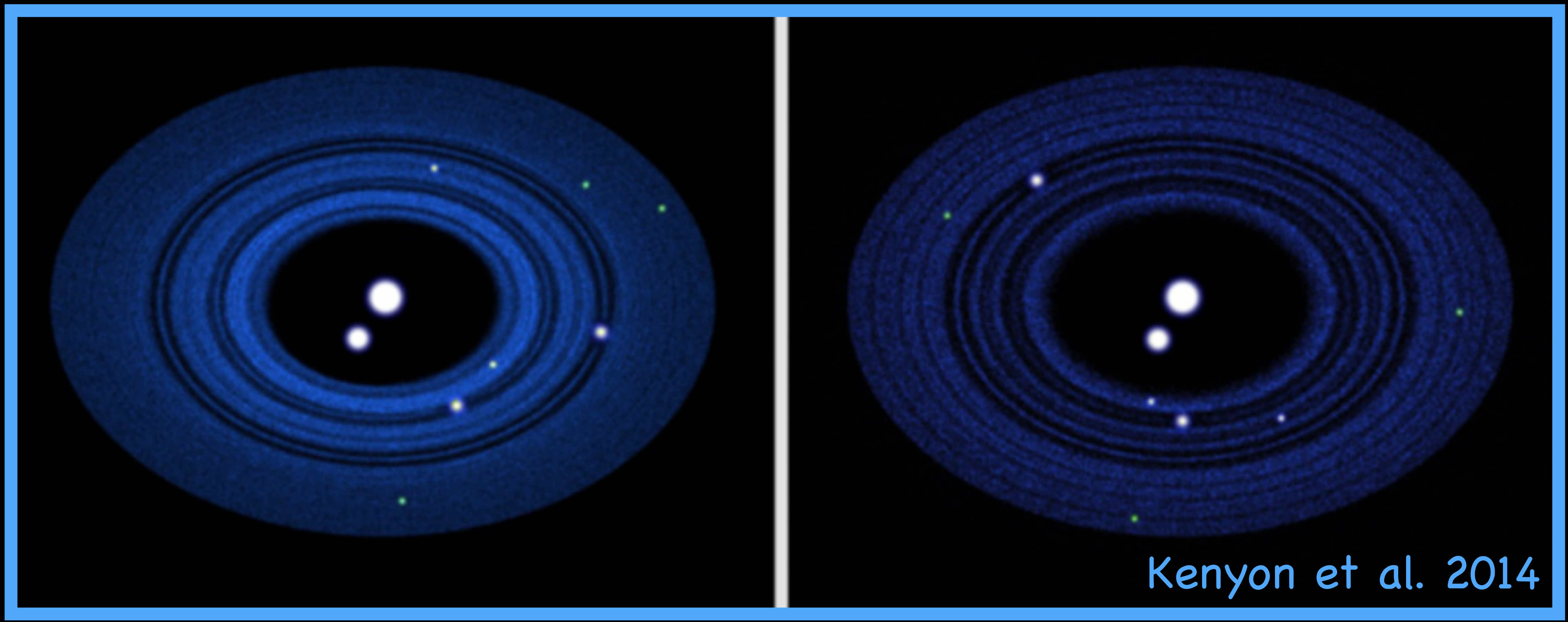


Point masses in binary (Pluto + Charon)
One resolved elongated body (Styx, Nix, or Kerberos)

Unidentified Spin resonances- Kerberos



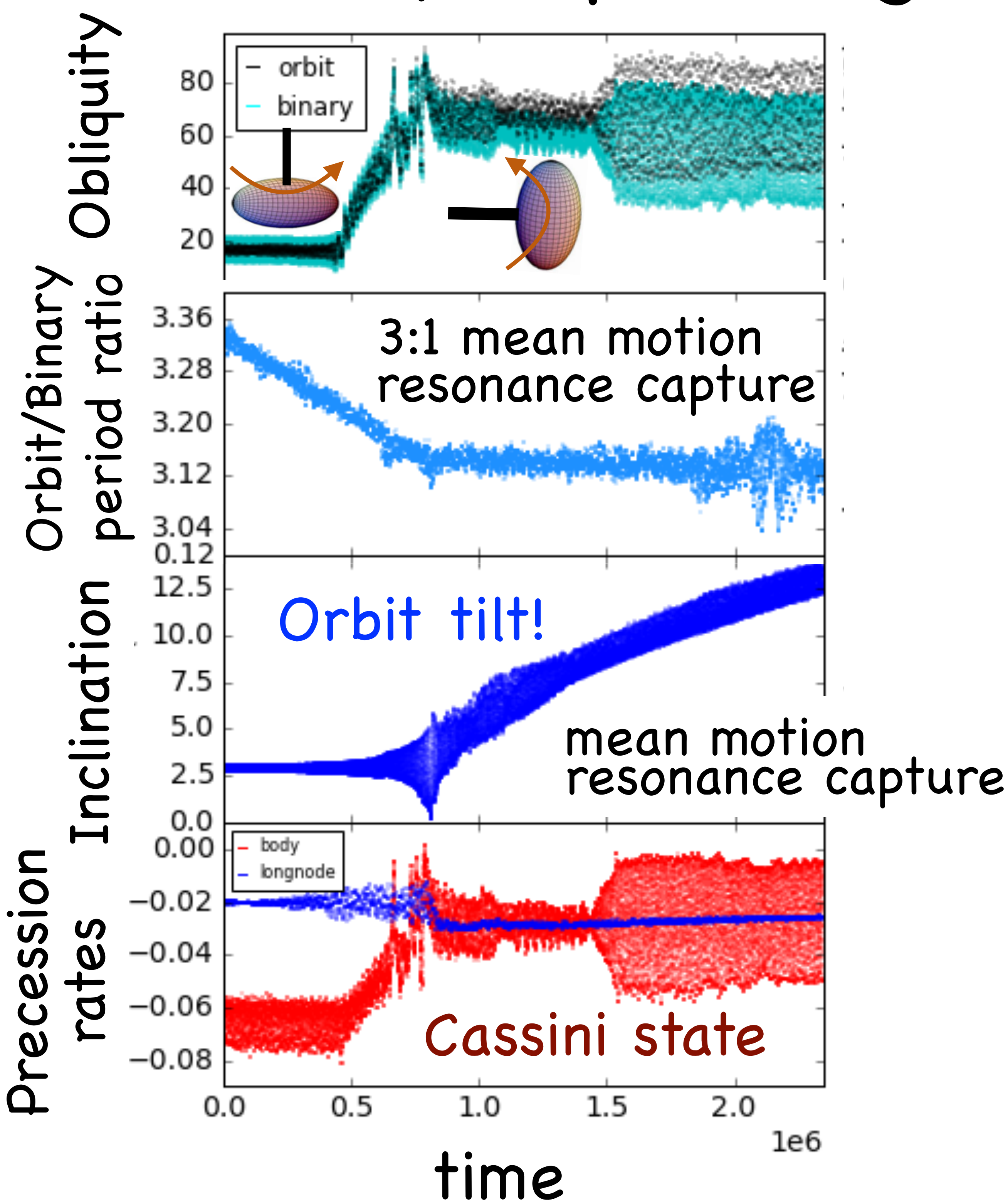
Formation of Pluto Satellite System



Post collision 6 bodies alone under tidal evolution causes instability and cannot explain current near resonant configuration of satellites (Cheng+2014)

The system could have formed from and **evolved** in a circumbinary disk (Kenyon+2014) orbital drift in past!

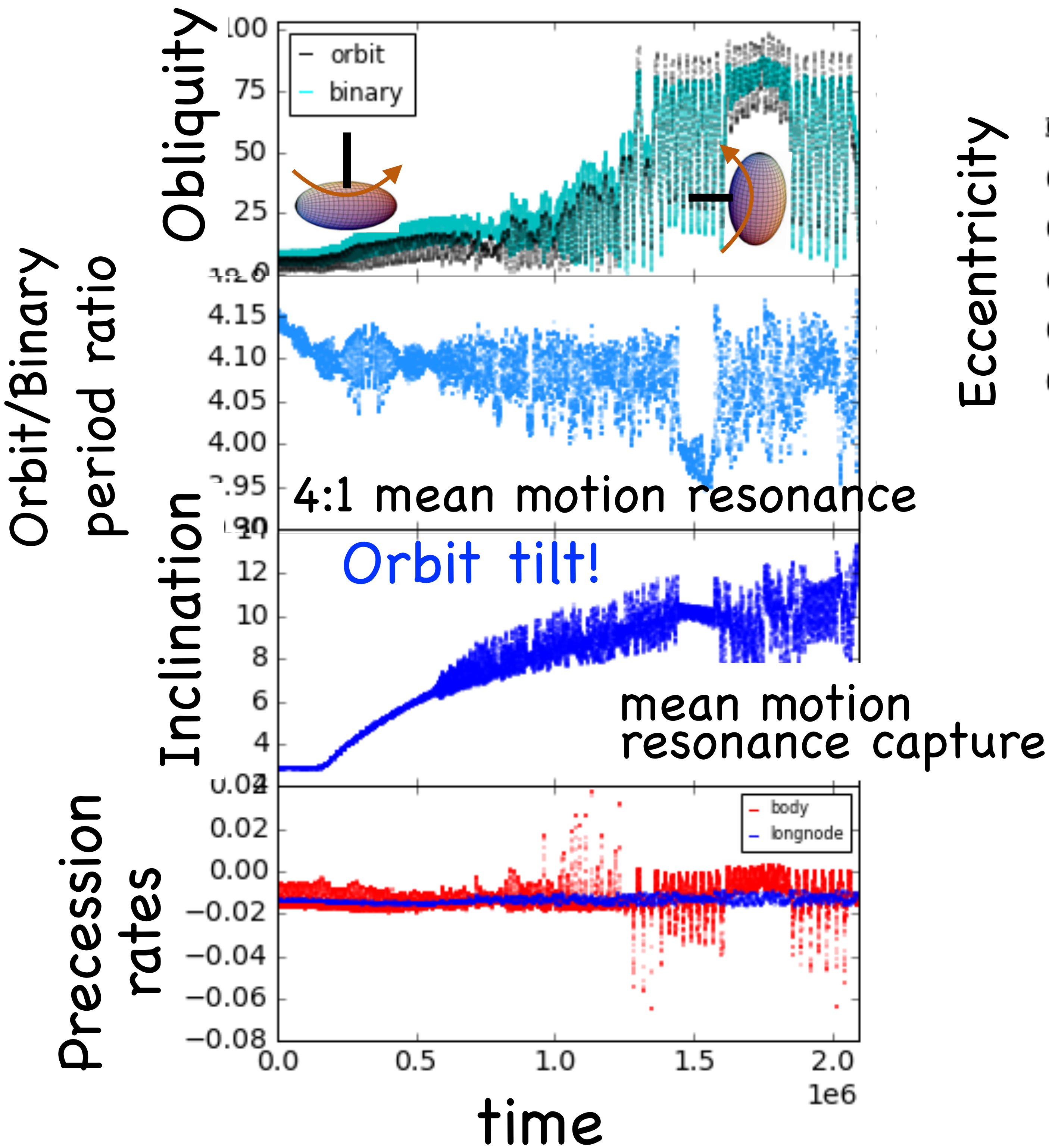
Circumbinary Obliquity evolution with slowly separating binary - Styx

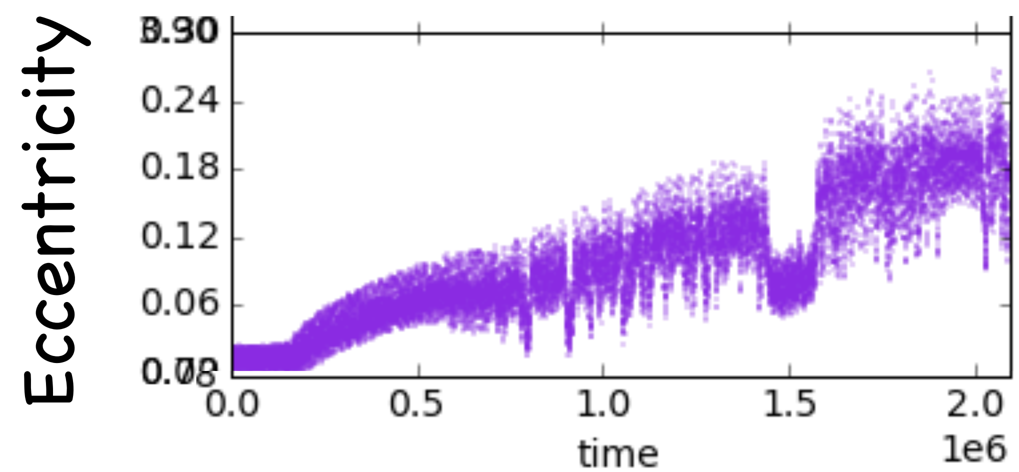


Inclination excited in 3:1 mean motion resonance

Obliquity increase happens just before capture into mean motion resonance

Circumbinary Obliquity evolution with slowly separating binary - Nix





mean motion resonance +lifting obliquity also happens for faster spinning Nix!

Orbital Resonance

Ratio of two periods is near an integer

$$3P_{Charon} \approx P_{Styx}$$

$$n_{Charon} \approx 3n_{Styx}$$ angular rotation rates

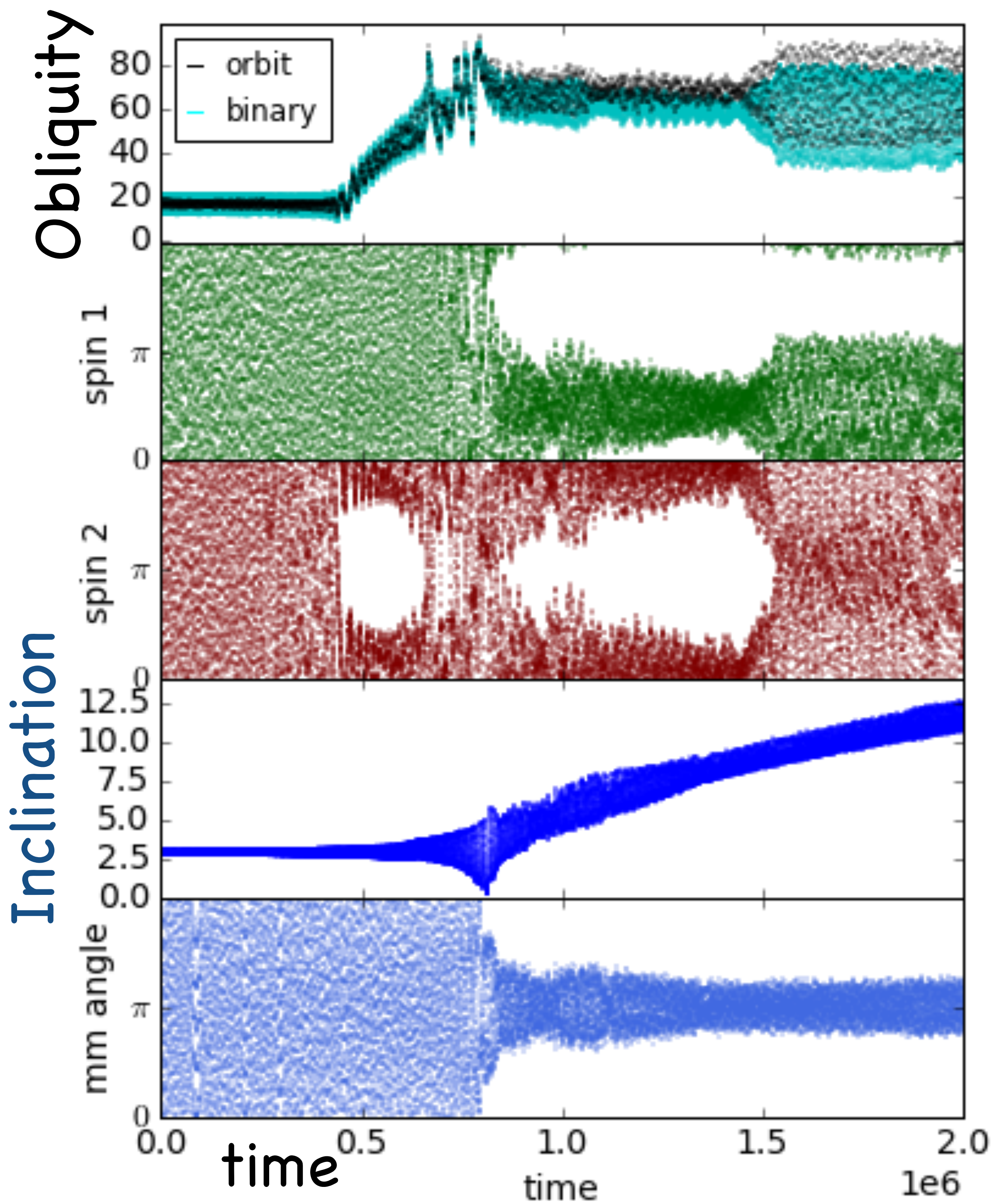
integrate

$$n = \frac{d\lambda}{dt}$$

$$\lambda_{Charon} - 3\lambda_{Styx} \approx \text{constant}$$

Resonant angle constant

Styx with separating binary



Resonant Angles

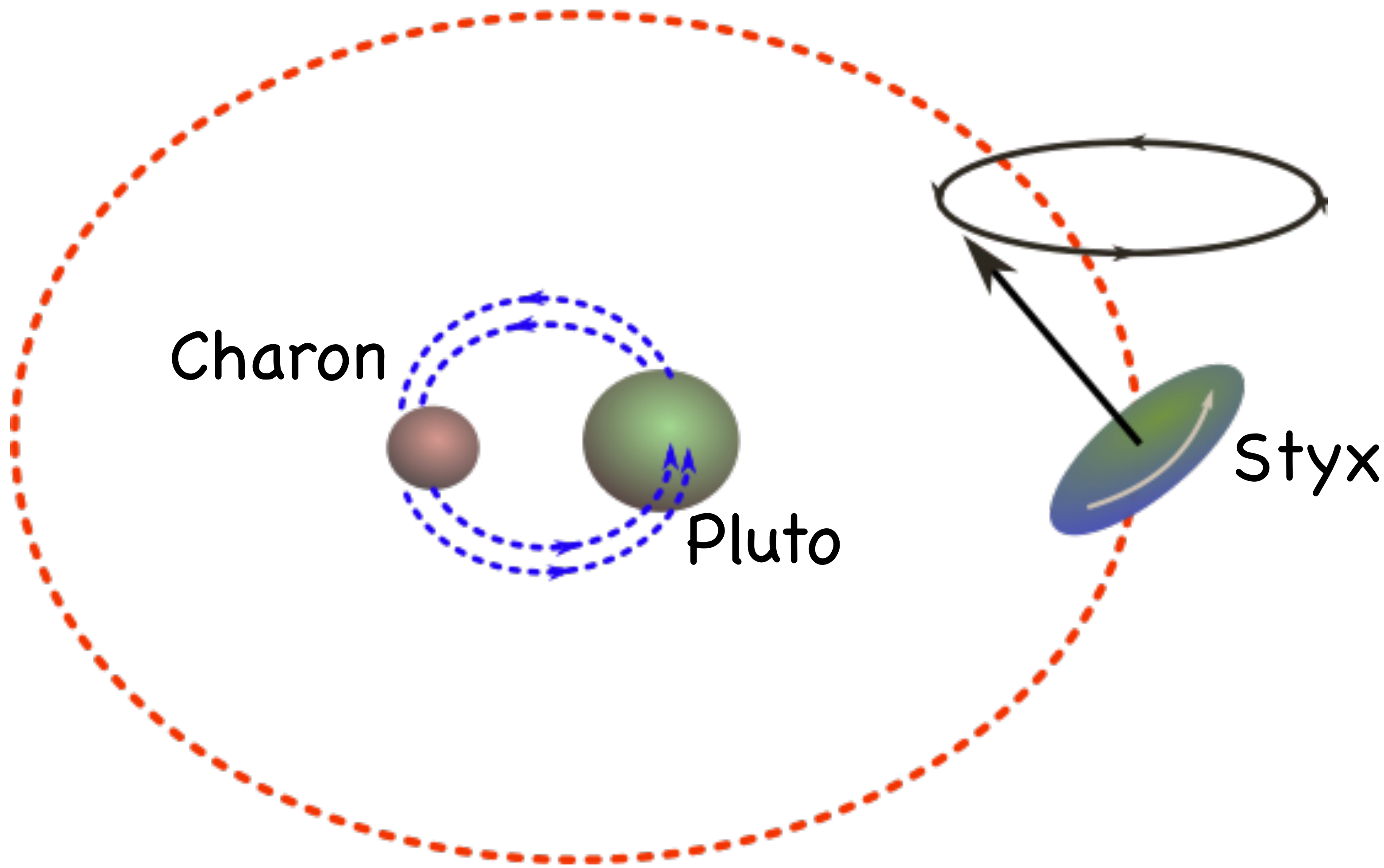
$3\lambda - \lambda_B - \Omega - \Omega_s$

$3\lambda - \lambda_B - 2\Omega_s$

Spin precession-mean-motion resonance

$3\lambda - \lambda_B - 2\Omega$

$$3\lambda - \lambda_B - 2\Omega_s = (\lambda - \lambda_B) + 2(\lambda - \Omega_s)$$



In the orbital frame moving with Styx
The binary appears to orbit twice
and Styx precesses once

DEMO

Summary

- Tidal spin down times are long for Pluto's satellites
- Satellites have not spun down
- In hindsight, this should have been expected/predicted
- Tidal evolution alone only explains Styx's obliquity via intermittent variations
- Outward migration of Charon causes capture into mean motion resonance and lifts obliquities
- A new resonant mechanism: Commensurability between spin precession rate and mean motion resonance angle
- Could flip spins for **all** of Pluto and Charon's satellites, explaining their high obliquities
- Obliquities need not be primordial
- Perhaps they were all previously in mean motion resonance with Pluto-Charon. Either system was unstable or obliquity flips took place while embedded in a disk

Motivations for developing theory for the spin-precession/mean-motion resonance

Uranus:

- Possibly previously in mean motion resonance with another giant planet during Nice model scenarios
- High obliquity current explained only via direct collision with a planet (Safronov'69, Parisi+08)
- Is there a non-collisional explanation for its high obliquity? see Boue+10 involving a close encounter

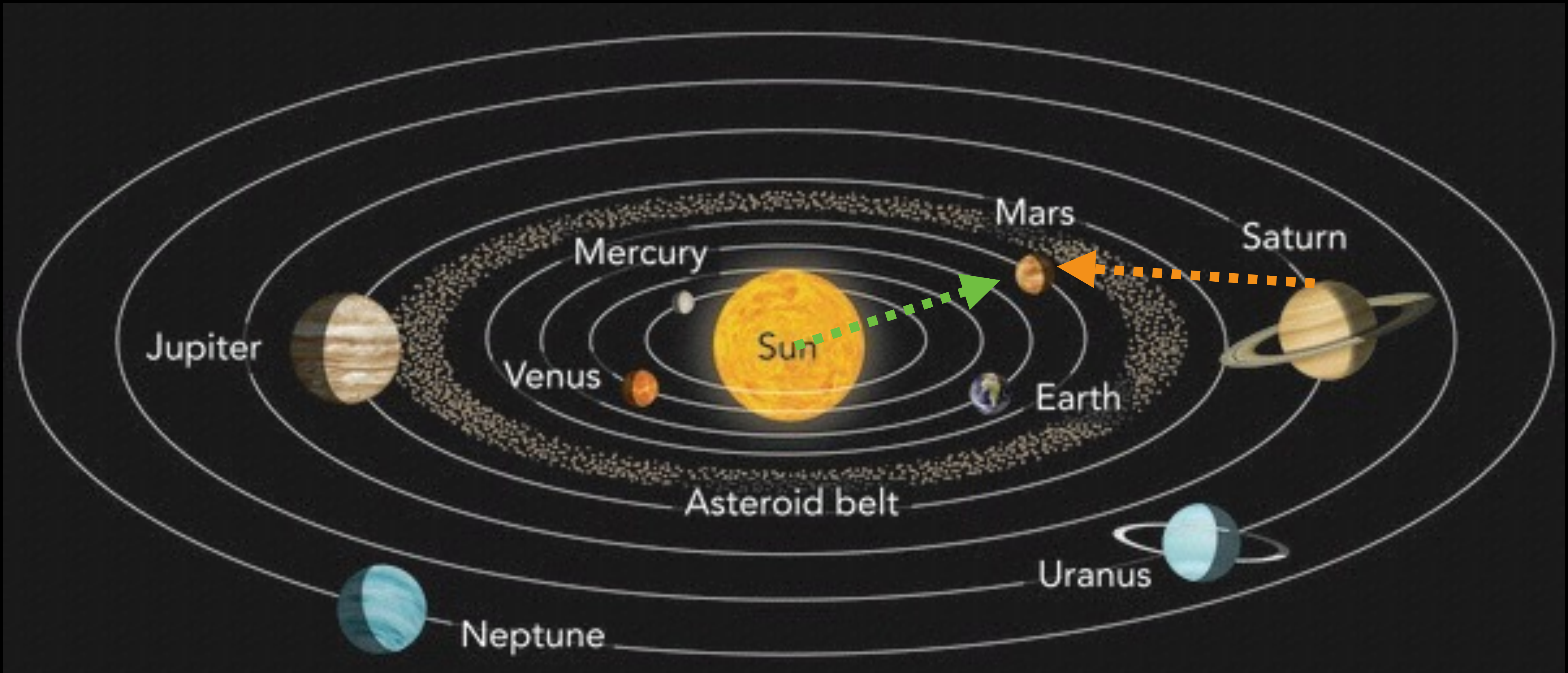
Exoplanets:

- Obliquity and spin-orbit resonance affect climates

Why was this resonance not previously studied?

Planets perturb Mars' orbit

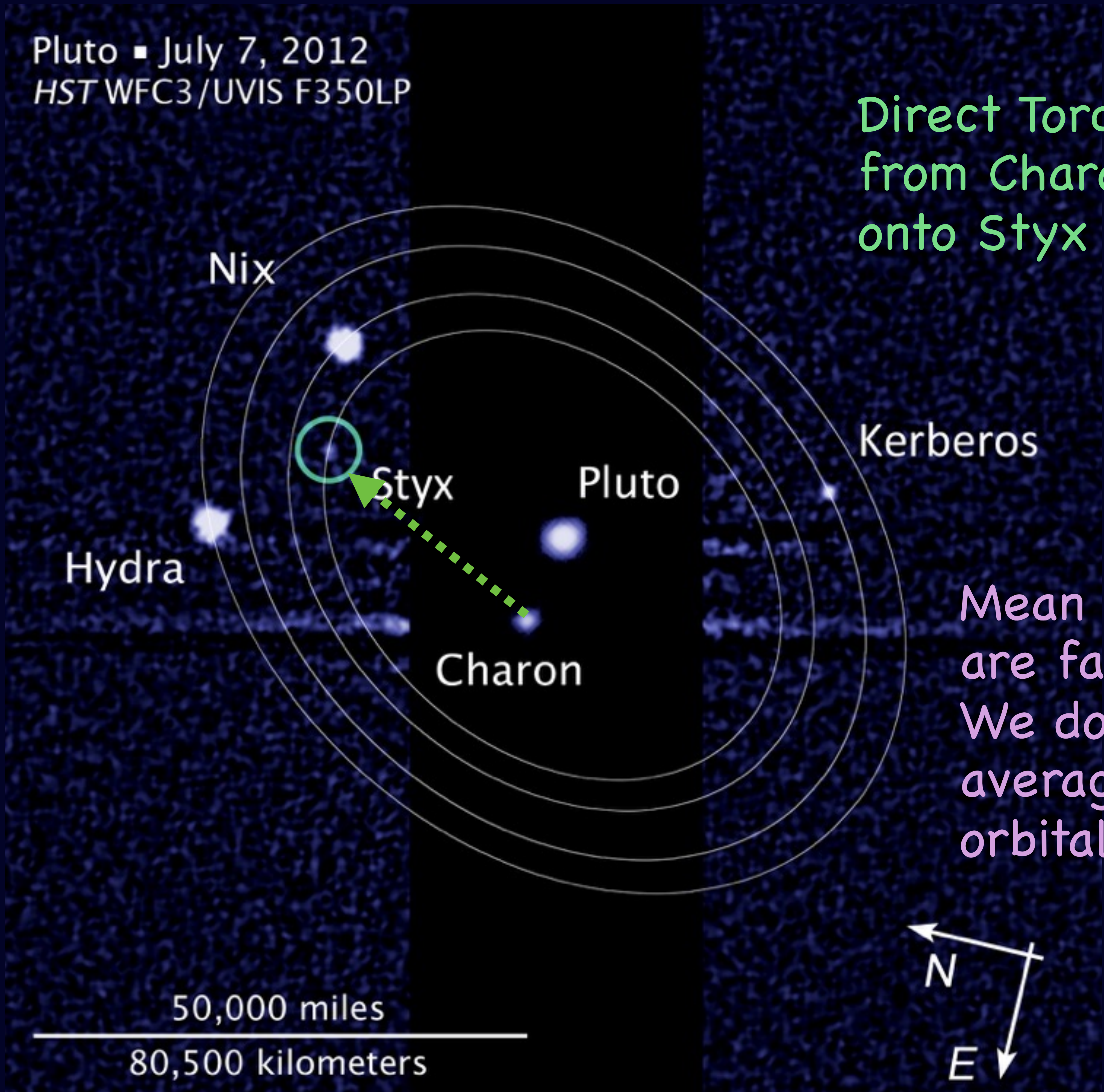
Torque from Sun causes obliquity changes on Mars



Precession is slow

Average over orbital period

Secular evolution



Direct Torque from Charon onto Styx or Nix

Mean motions are fast angles
We do not average over orbital period

Gravitational Potential interaction between a point mass and an extended mass

Quadrupole moment matrix
Moments of Inertia matrix
MacCullough's formula

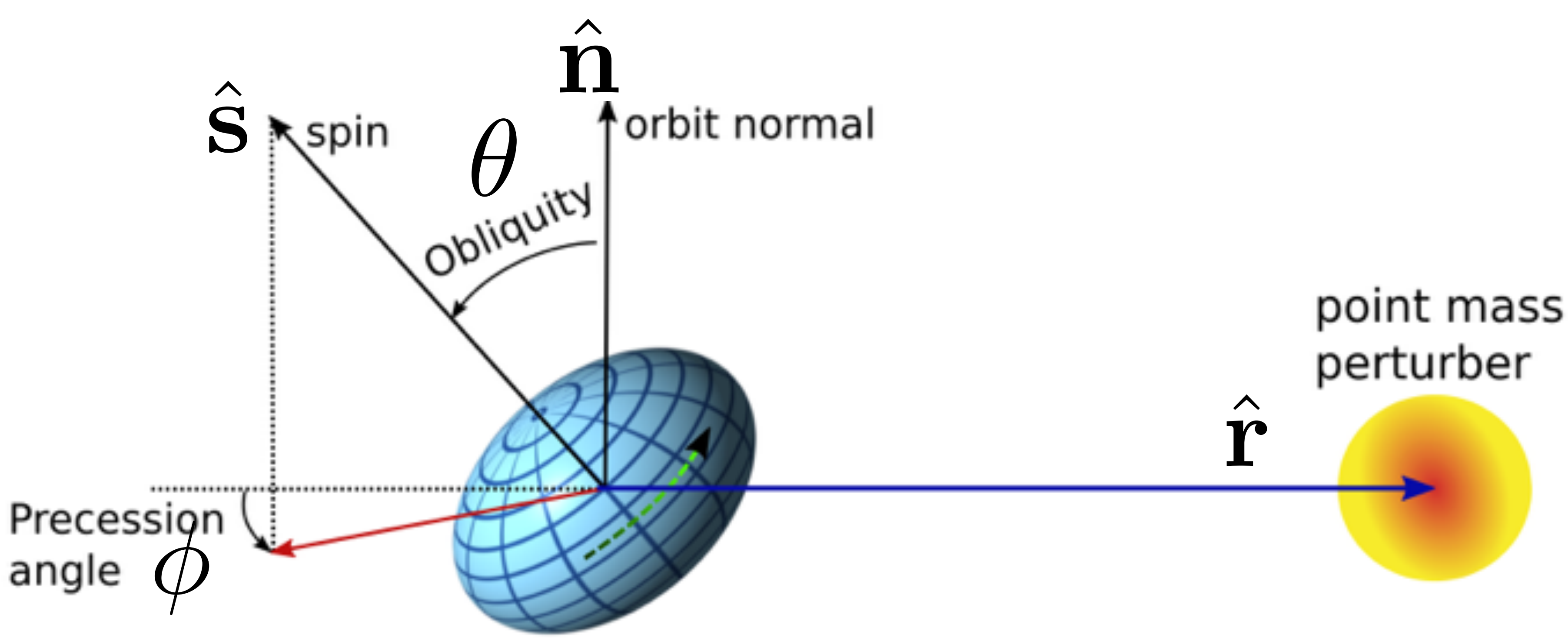
$$U_{interaction} = GM \int \frac{\rho(\mathbf{r}')d^3r'}{|\mathbf{r} - \mathbf{r}'|}$$

$$= 3(C - A)\frac{GM}{r^3}(\hat{\mathbf{s}} \cdot \hat{\mathbf{r}})^2$$

moments of inertia

Instantaneous torque $\mathbf{T} = 3(C - A)\frac{GM_*}{r^3}(\hat{\mathbf{r}} \cdot \hat{\mathbf{s}})(\hat{\mathbf{r}} \times \hat{\mathbf{s}})$

spin axis is axis of symmetry $\hat{\mathbf{s}} = (\cos\phi\sin\theta, \sin\phi\sin\theta, \cos\theta)$



Toward a theory for spin-precession mean-motion resonance

Unperturbed Hamiltonian: precession via central star
Perturbation from another orbiting object

$$H(p, \phi, t) = H_0(p, \phi) + H_1(p, \phi, t)$$

We do not average over the orbit of the perturber
We use MacCullough's formula to expand the potential perturbation in orbital elements

$$H_1(p, \phi, t) = \frac{3(C-A)}{Cw} \frac{G m_p}{|\mathbf{r} - \mathbf{r_p}|^5} \frac{((\mathbf{r} - \mathbf{r}_p) \cdot \hat{\mathbf{s}})^2}{2}$$

Averaging over orbit

essentially replace radial vector with orbit normal

$$\frac{d\hat{\mathbf{s}}}{dt} = \alpha_s (\hat{\mathbf{s}} \cdot \hat{\mathbf{n}})(\hat{\mathbf{s}} \times \hat{\mathbf{n}})$$

precession direction now depends on orbit normal

$$\alpha_s = \frac{3n^2}{2} \frac{(C - A)}{Cw}$$

precession rate

Orbit normal can be time dependent: Models for obliquity evolution of Mars, Saturn, evolution to Cassini state (Gladman, Ward, Touma, Laskar, Columbo...)

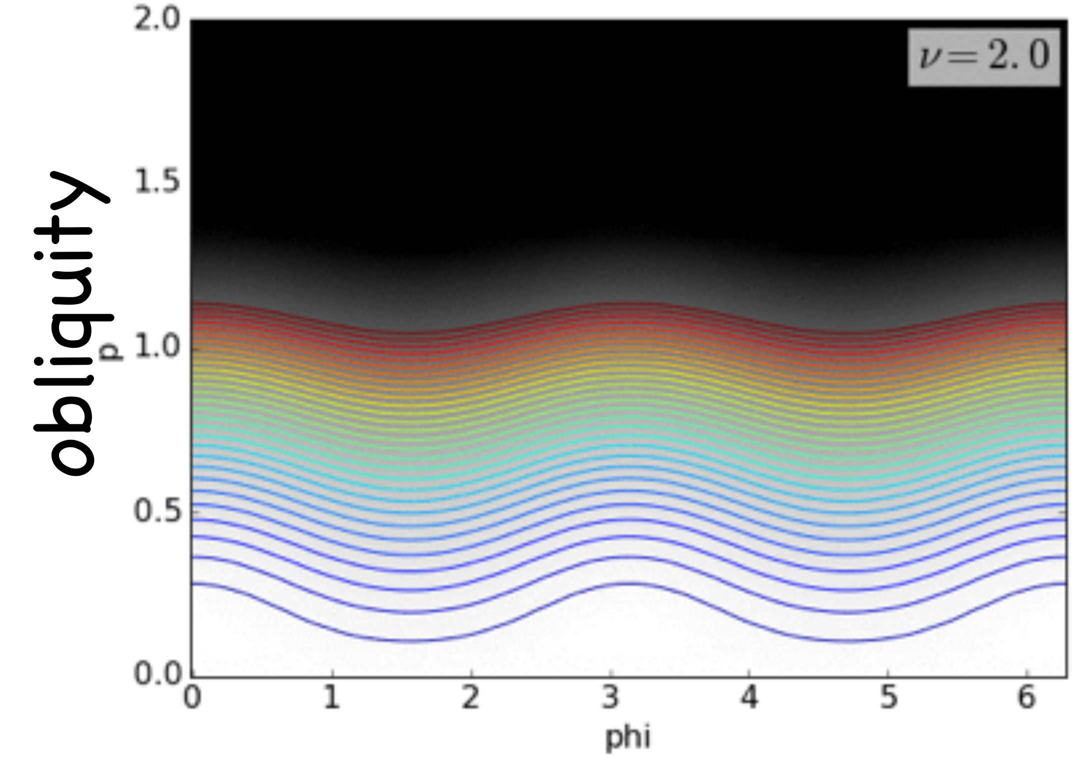
Energy $= \frac{\alpha_s}{2} (\hat{\mathbf{s}} \cdot \hat{\mathbf{n}})^2$

equation of motion can be derived from a Hamiltonian

$$H(p, \phi) = \frac{\alpha_s}{2} (1 - p)^2 \qquad p = (1 - \cos\theta)$$

gyroscopic approximation gives a Hamiltonian with obliquity conjugate to precession angle

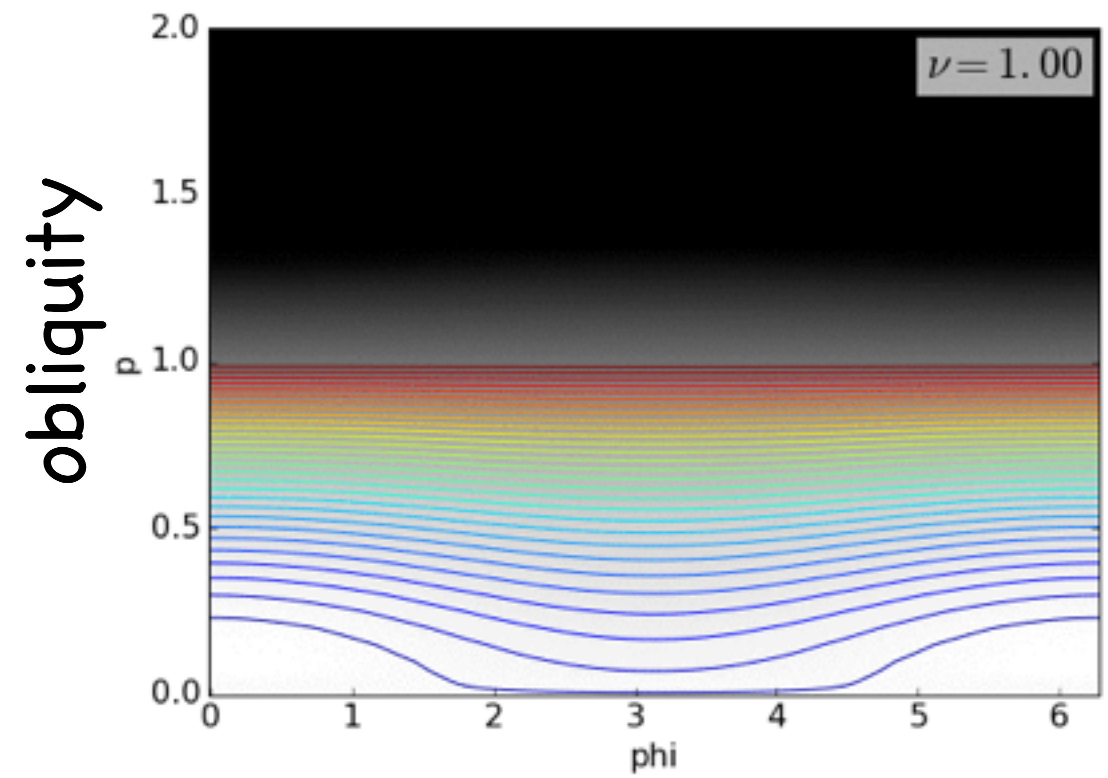
Shapes of Hamiltonian as a function of distance to resonance



distance to resonance

$$K(p,\varphi,\tau) = \frac{1}{2}(p-1)^2 + \frac{1}{2}\nu p$$
$$-\epsilon p(2-p)\cos(2\varphi)$$

$$K(p,\varphi,\tau) = \frac{1}{2}(p-1)^2 + \nu p$$
$$-\epsilon(1-p)\sqrt{p(2-p)}\cos\varphi$$



precession angle

Application to Styx/Nix

Mass ratio Charon/Pluto is 0.1
Styx/Nix/Kerberos/Hydra are not round, so precession rates are of order a thousand orbital periods.
Orbital periods are days.

-> Spin/precession mean motion resonance is strong and fast, consistent with what is seen in our simulations

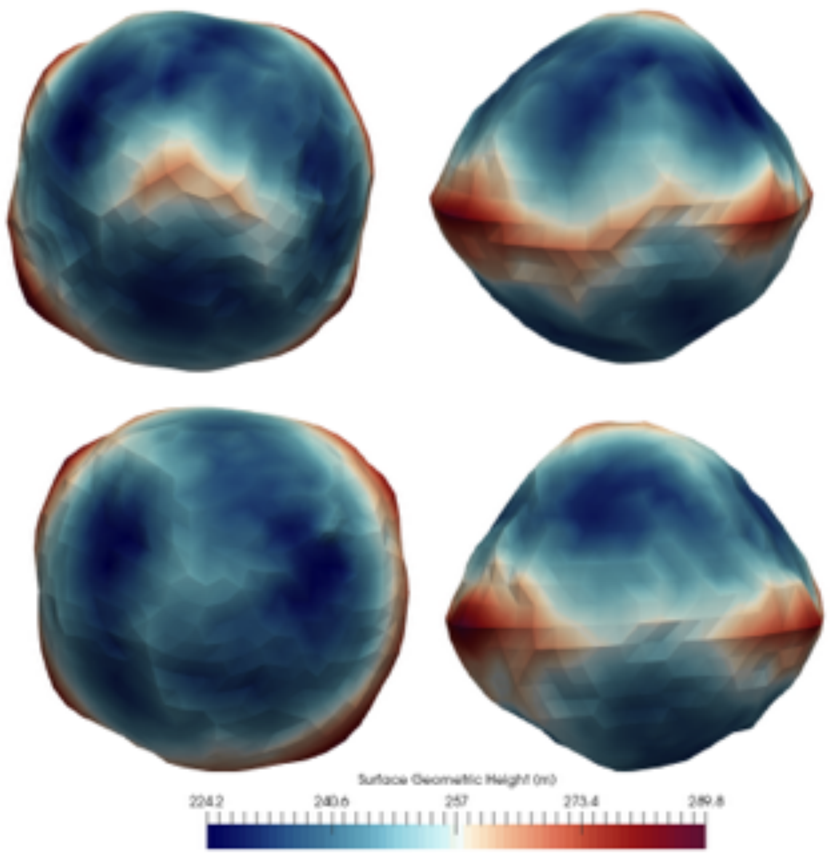
Styx and Nix spin resonances are low order in inclination or eccentricity

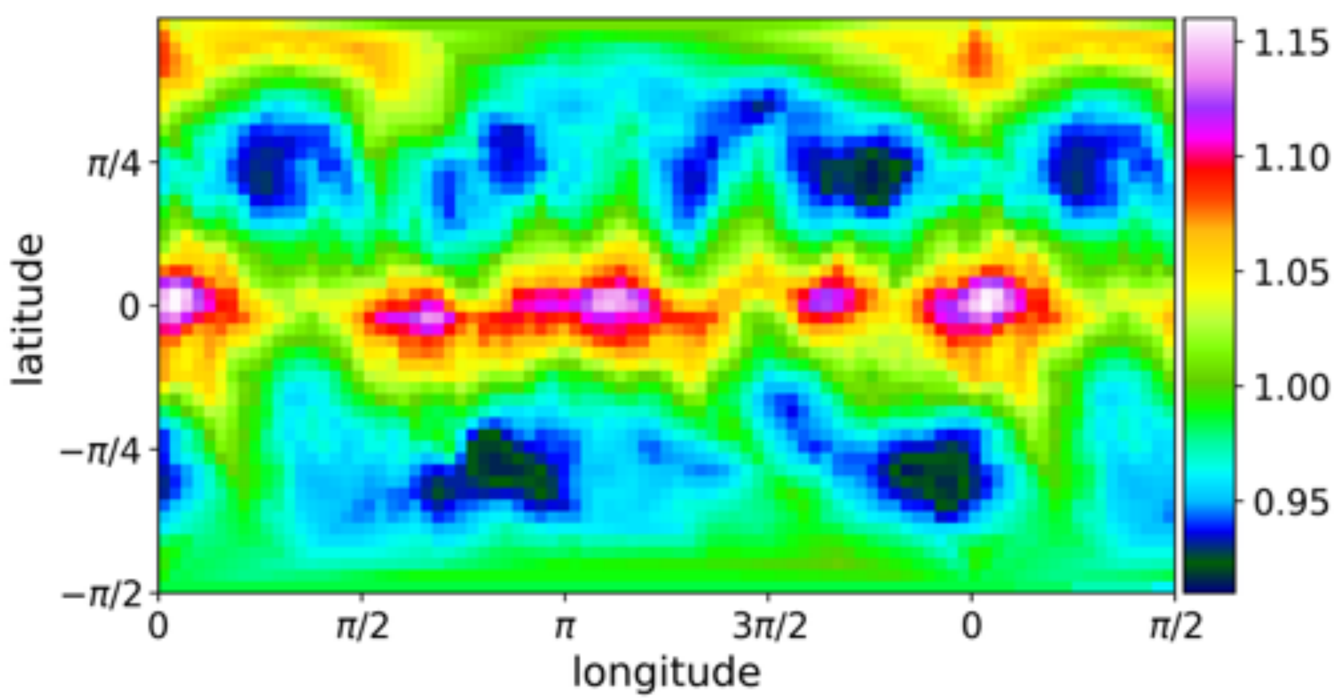
Kerberos, Hydra are higher order, still some question whether this resonance would be effective

Summary

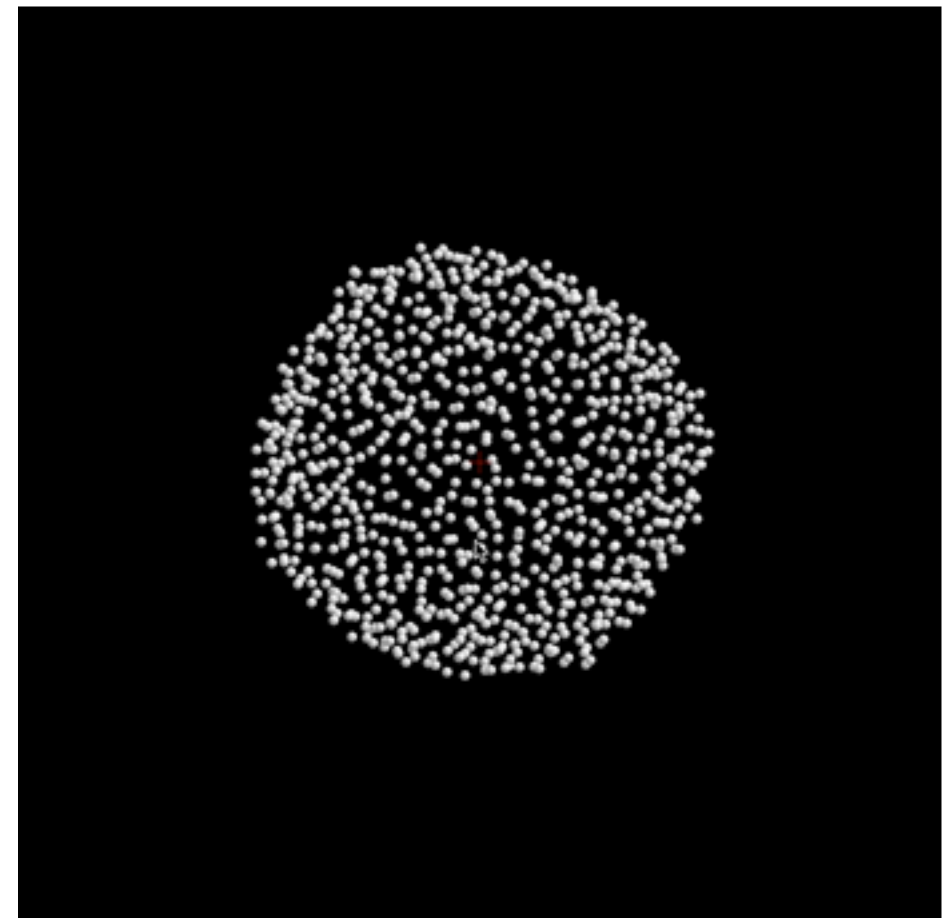
- We found a Hamiltonian theory for spin precession/mean motion resonance
- Low order terms in inclination and eccentricity are about as strong as secular spin resonances
- Theory is relevant for tilting of Pluto+Charon's satellites
- This resonance does not work for Uranus (too slow) but could be important during spin down of exoplanets
- This is a spin resonance discovered from numerical simulations

The Shape of Asteroid Bennu

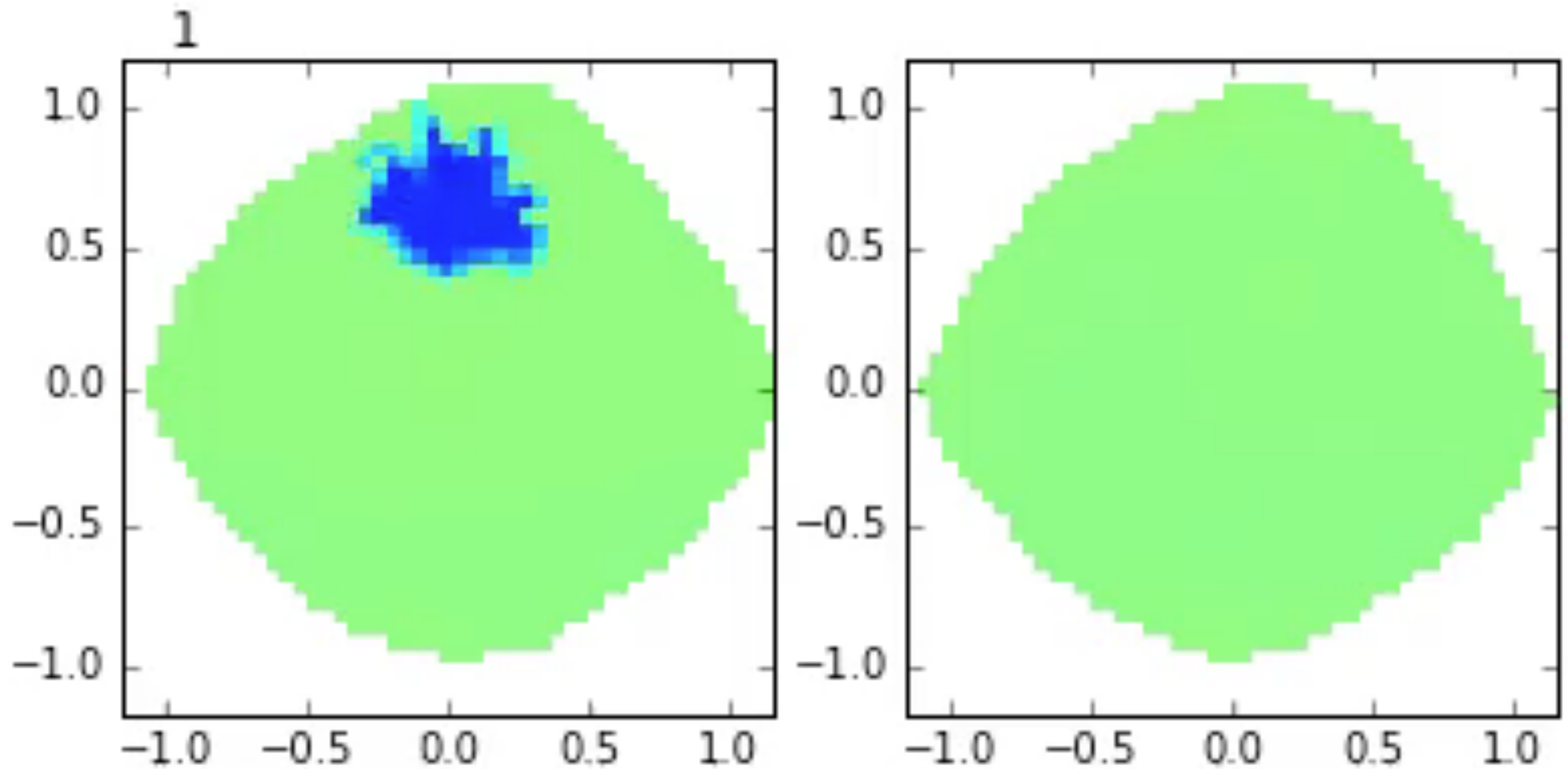




Bennu shape model in our simulation



Impacts modeled with the mass/spring model



Seismic source:

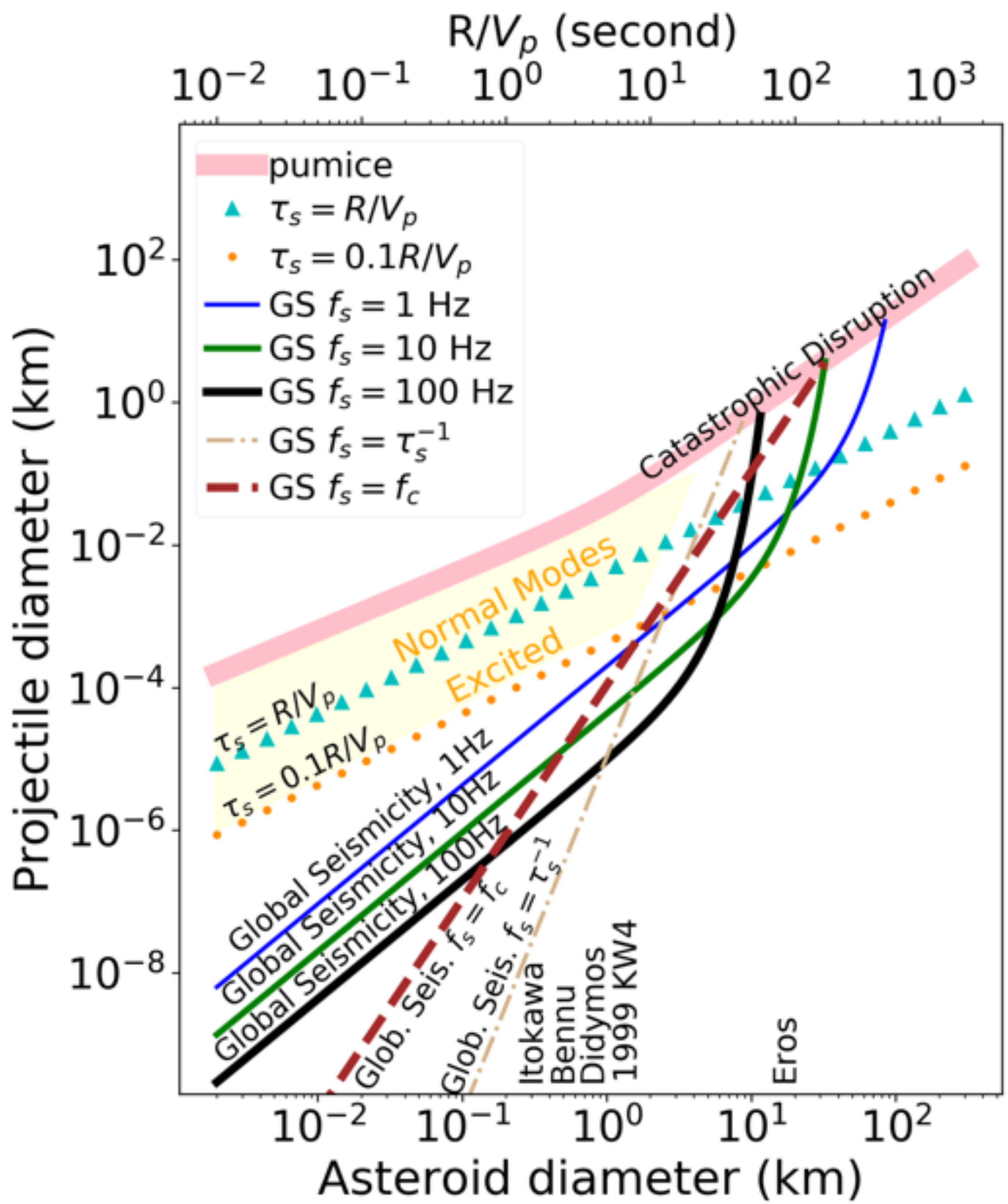
Radial force exerted on surface particles.

Depends on a source time

Force amplitude

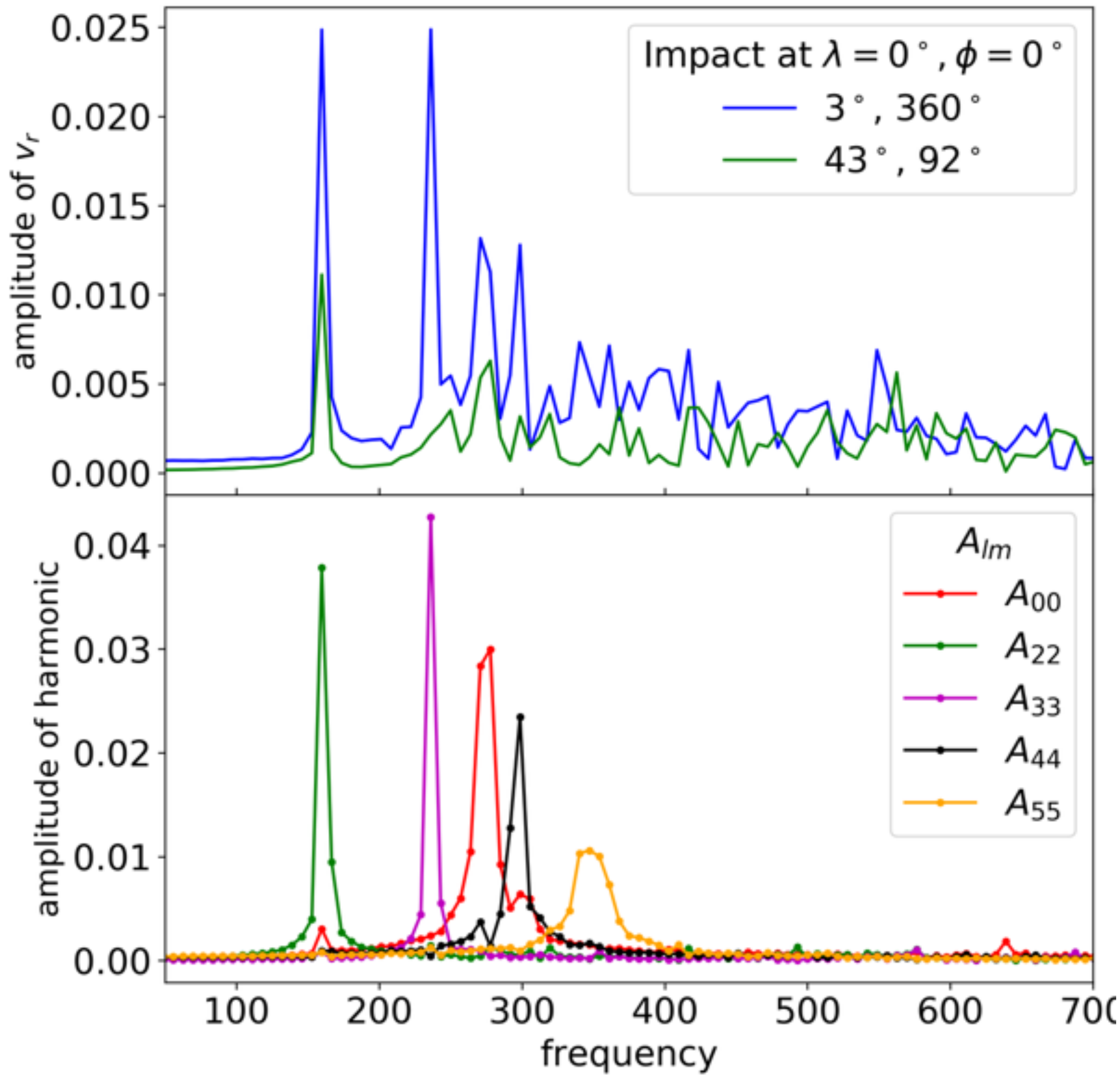
Area of surface where applied

Parameters for these derived via scaling using a seismic efficiency (energy), amplification factor (momentum) and crater size estimates



Regime for excitation of low frequency Normal modes by Impacts

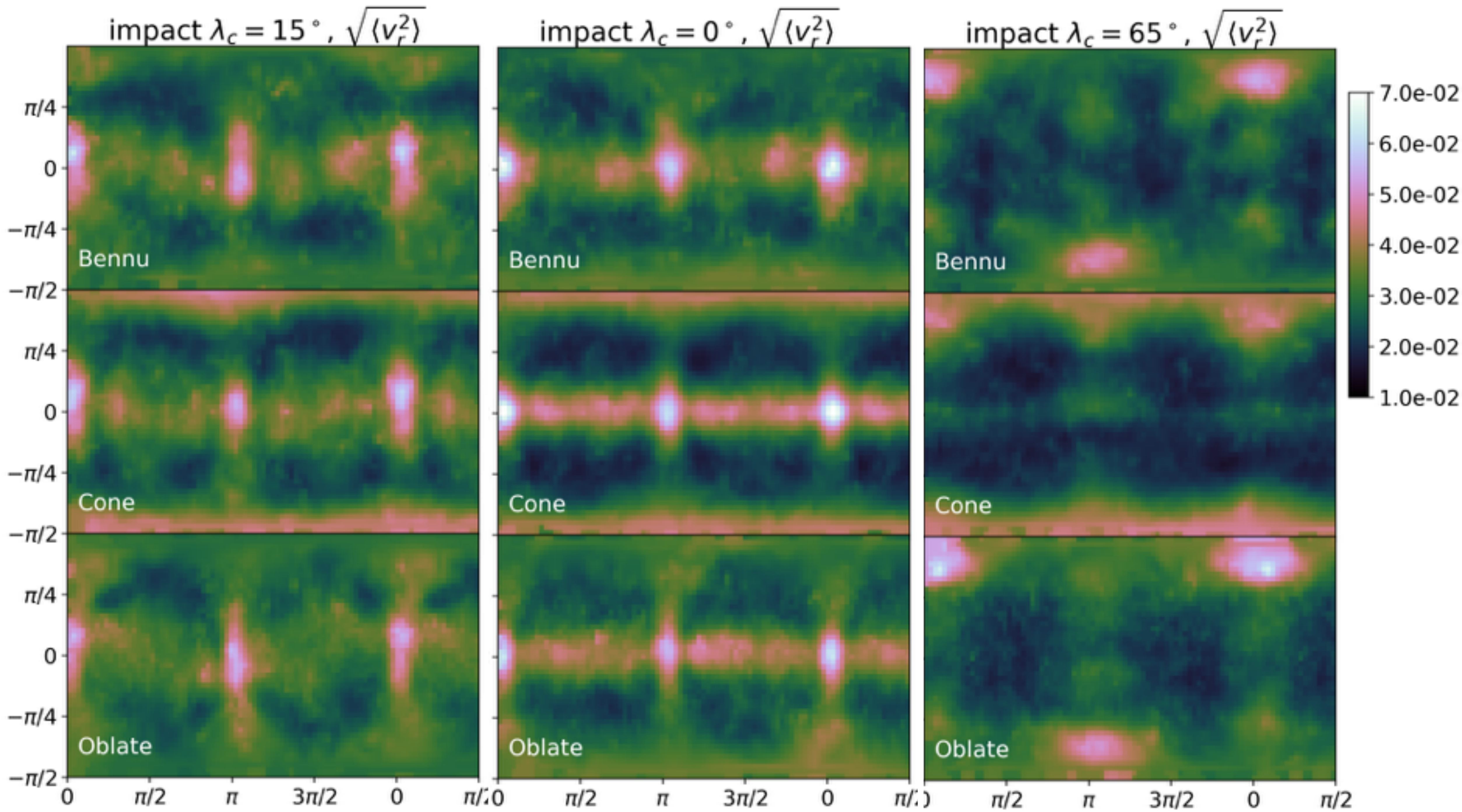
Normal modes identified in the simulation



Checks on code:

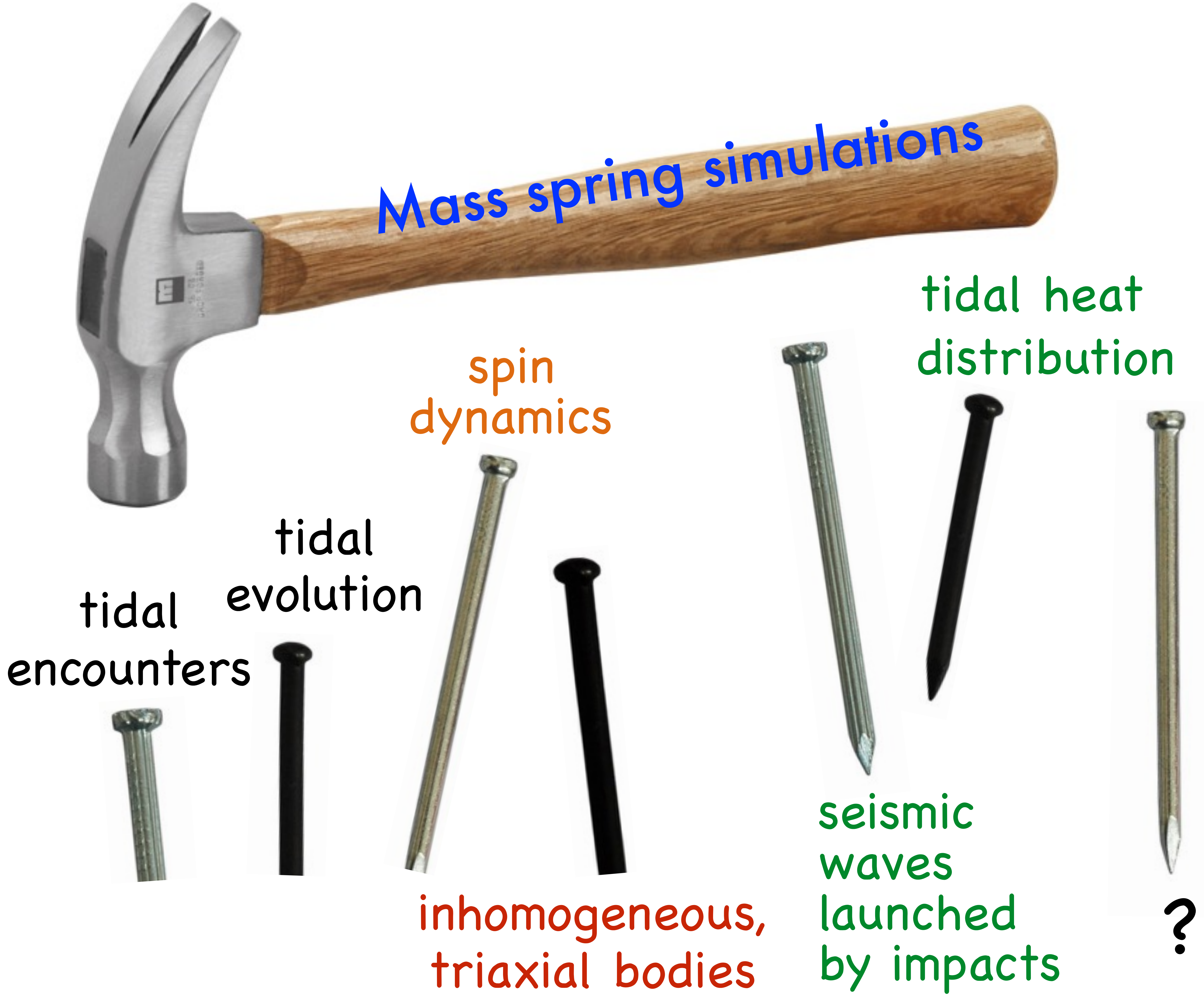
- Travel time for antipodal focus consistent with estimated Rayleigh wave speed
- Normal mode frequencies are consistent with 10% of predicted for a isotropic homogeneous elastic sphere

Distribution of vibrational energy following the impact

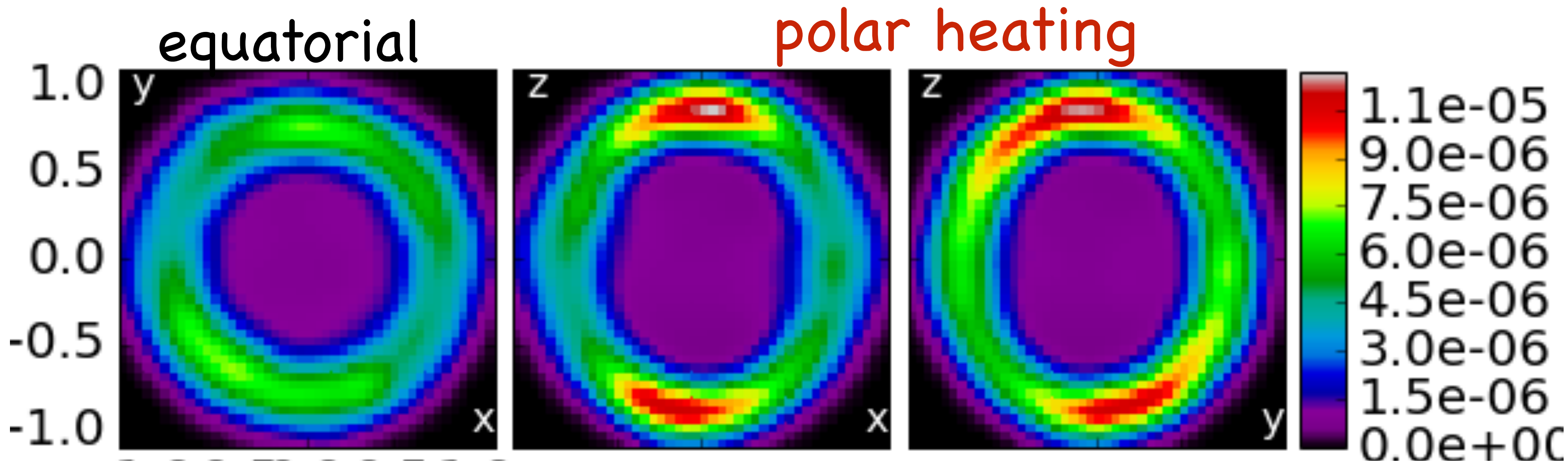


Summary

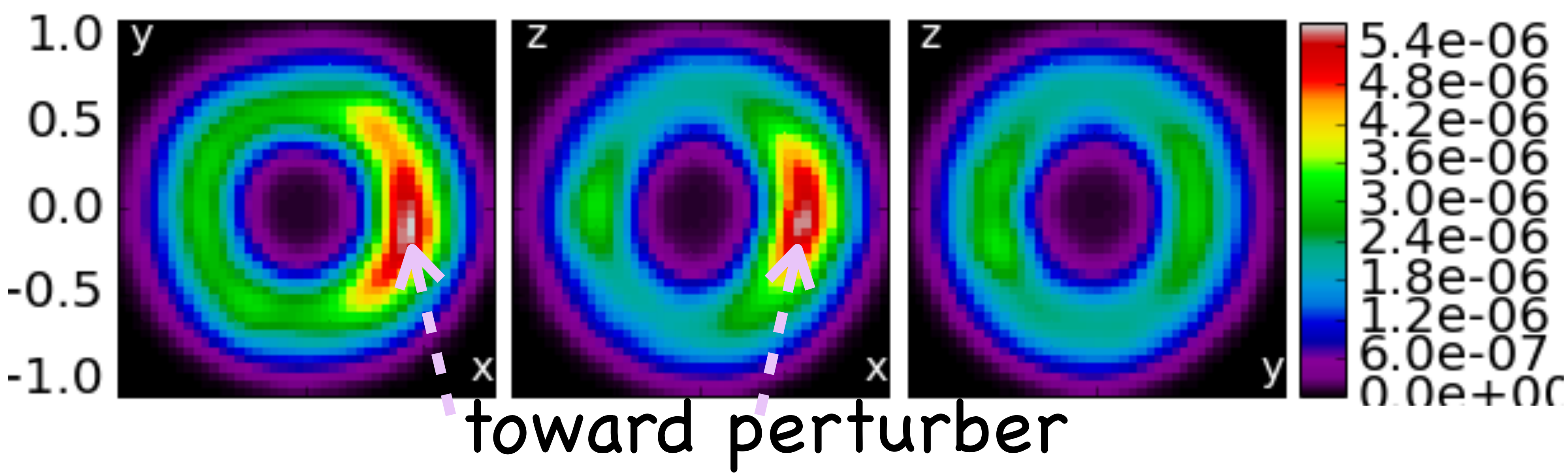
- Rare large impacts could excite low frequency seismic waves on small asteroids.
- Low frequency normal modes are excited
- The distribution of seismic vibrational energy is not even
- Vibrational energy primarily strong at impact point and its antipode
- We primarily would expect slumping toward an equatorial ridge from these two points
- The 4 peaks on Bennu's equatorial ridge is not explained


Mass spring simulations
tidal heat distribution
spin dynamics
tidal evolution
tidal encounters
seismic waves launched by impacts
inhomogeneous, triaxial bodies
?

Energy dissipation in the damped springs



A hard but dissipative spherical crustal shell on top of a soft but lower viscosity interior. In an eccentric orbit about a central star and tidally locked. Heating rate per unit volume in a plane that bisects the body with xy the orbital plane and z the orbital angular momentum axis.



A tidally locked planet that is only a few stellar radii from the star. Due to the proximity of the star the internal heating is not symmetric. The crust facing the star should be thinner and vulcanism would be more likely on this side of the planet. These simulations illustrate that a coupled heat transfer and tidal model would predict aspherical internal structure, and possibly episodes of vulcanism and uneven crustal thickness that would enhance capture into spin-orbit resonance.

Type of Spin Resonance	Integer relations	Consequences
Spin orbit $w = \frac{k}{2} n$	orbital mean motion, spin (Wisdom, Goldreich, Peale)	Regular satellites as they tidally spun down passed through resonance Capture: Mercury Tumbling: Hyperion
Spin secular $w = \dot{\Omega}_{eigen}$	spin precession rate, orbit precession rate (Laskar, Touma, Ward & Hamilton)	Obliquity variation: Climate variations Mars, Tilting Saturn
Spin binary $w - n = \frac{k_B}{2}(n_B - n)$	orbital mean motion, binary mean motion, spin Correia+16	Pluto/Charon satellites instability?

Tidal Spin down of Pluto/Charon's minor satellites

– Obliquity increase is caused by tidal evolution prior to tidal lock (Goldreich79)

Obliquity increase is slow, at the same speed as spin down. If bodies are not spun down to near spin synchronous then non-resonant tidal obliquity evolution could not have taken place

– Spin-orbit resonance can increase obliquity

High order spin-orbit resonances depend on eccentricity to a high power and so are irrelevant at the high spins of Pluto and Charon's minor satellites

– Spin states are more complex near a binary

Showalter and Hamilton 16, and Correia+16 would have been correct about tumbling if the spins were slower

Period Ratios

Weaver+2016	Orbit/Spin	Orbit/Binary
Styx	6	3.1566
Nix	13	3.8913
Kerberos	6	5.0363
Hydra	88	5.9810

near mean motion resonances

- Tidal spin-down has not happened
- Spins are too fast for spin/orbit resonances to be important
- Tidal obliquity evolution is too slow to have caused high obliquities

Toward a theory for spin-precession mean-motion resonance

$$H_1(p,\phi,t) = \frac{3(C-A)}{Cw}\frac{Gm_p}{|\mathbf{r}-\mathbf{r_p}|^5}\frac{((\mathbf{r}-\mathbf{r}_p)\cdot\hat{\mathbf{s}})^2}{2}$$

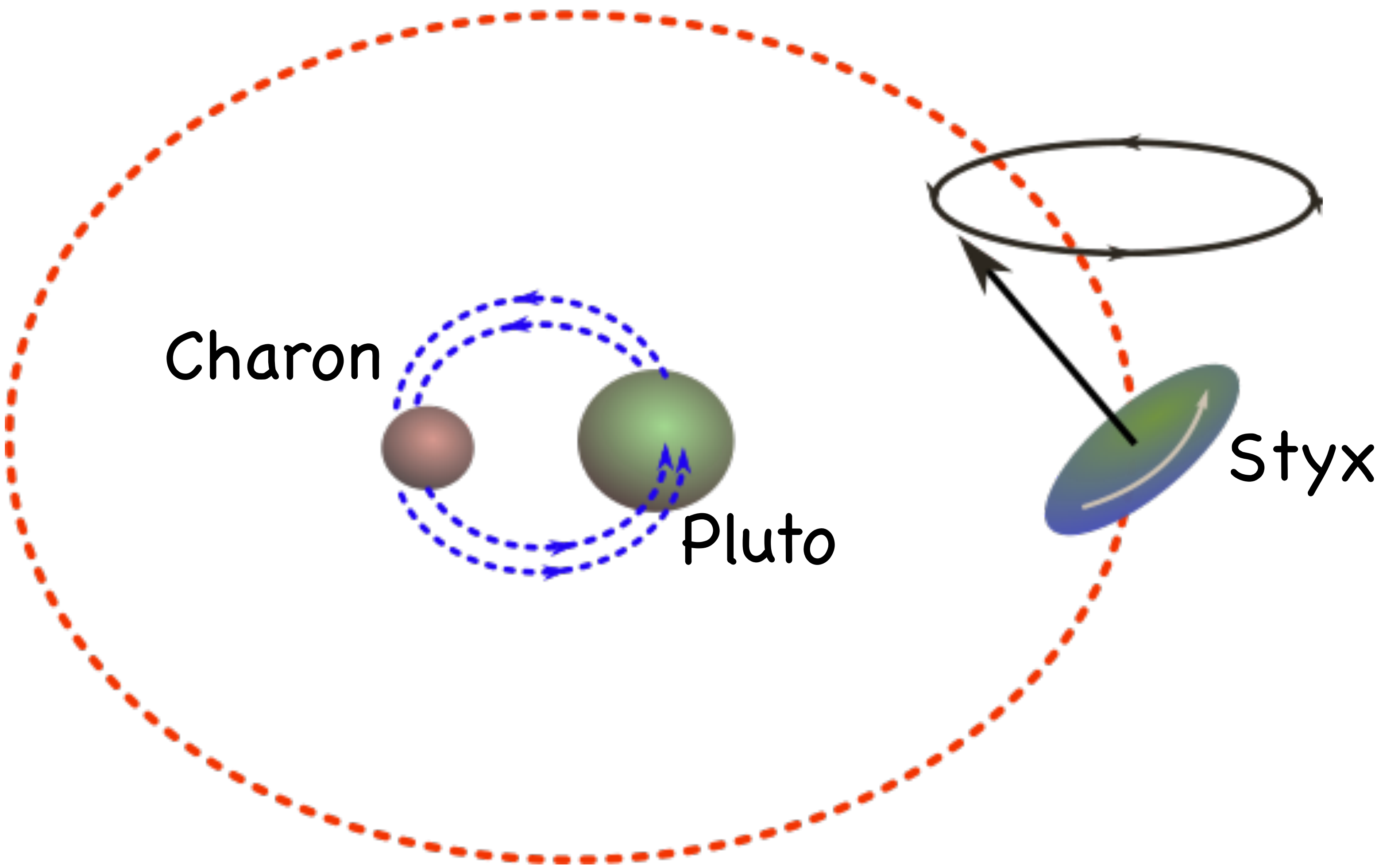
To zero-th order in eccentricity but first order in inclination with I ~s/2

$$\frac{x}{r} \approx \frac{x}{a} \approx \cos(\omega+\Omega+M) \approx \cos\lambda$$
$$\frac{y}{r} \approx \frac{y}{a} \approx \sin(\omega+\Omega+M) \approx \sin\lambda$$
$$\frac{z}{r} \approx \frac{z}{a} \approx 2s\sin(\omega+M) \approx 2s\sin(\lambda-\Omega).$$

and likewise for x_p, y_p, z_p

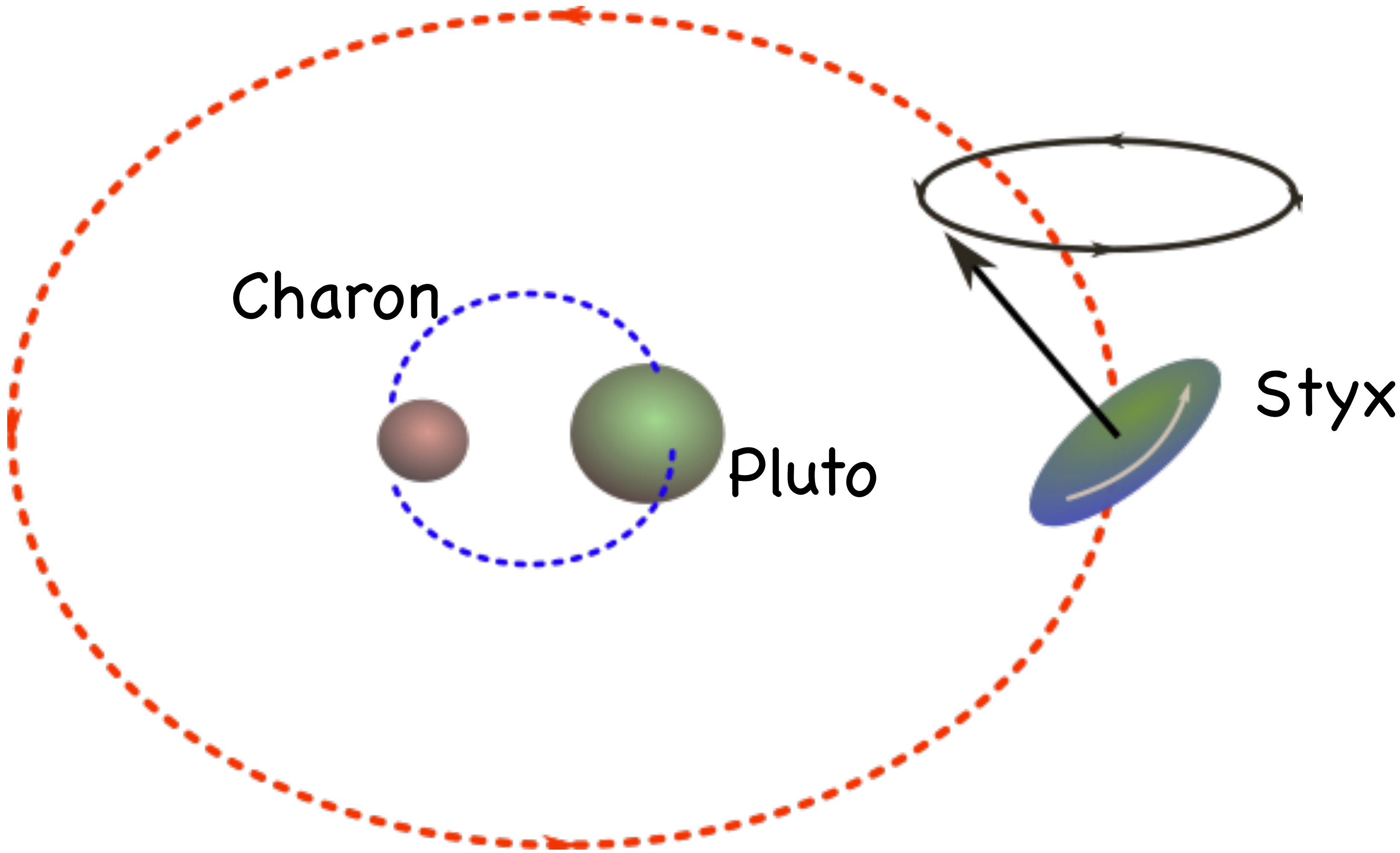
Expand $|\mathbf{r} - \mathbf{r}_p|$ in terms of Laplace coefficients as if you were doing a standard Celestial mechanics expansion of the disturbing function. Multiply all trig functions.

$$3\lambda - \lambda_B - 2\Omega_s = (\lambda - \lambda_B) + 2(\lambda - \Omega_s)$$



In the orbital frame moving with Styx
The binary appears to orbit twice
and Styx precesses once

$$3\lambda - \lambda_B - 2\Omega_s = 3(\lambda - \lambda_B) - 2(\Omega_s - \lambda_B)$$



In the orbital frame moving with Charon
Styx orbits twice as it precesses three times.

DEMO !!!!!

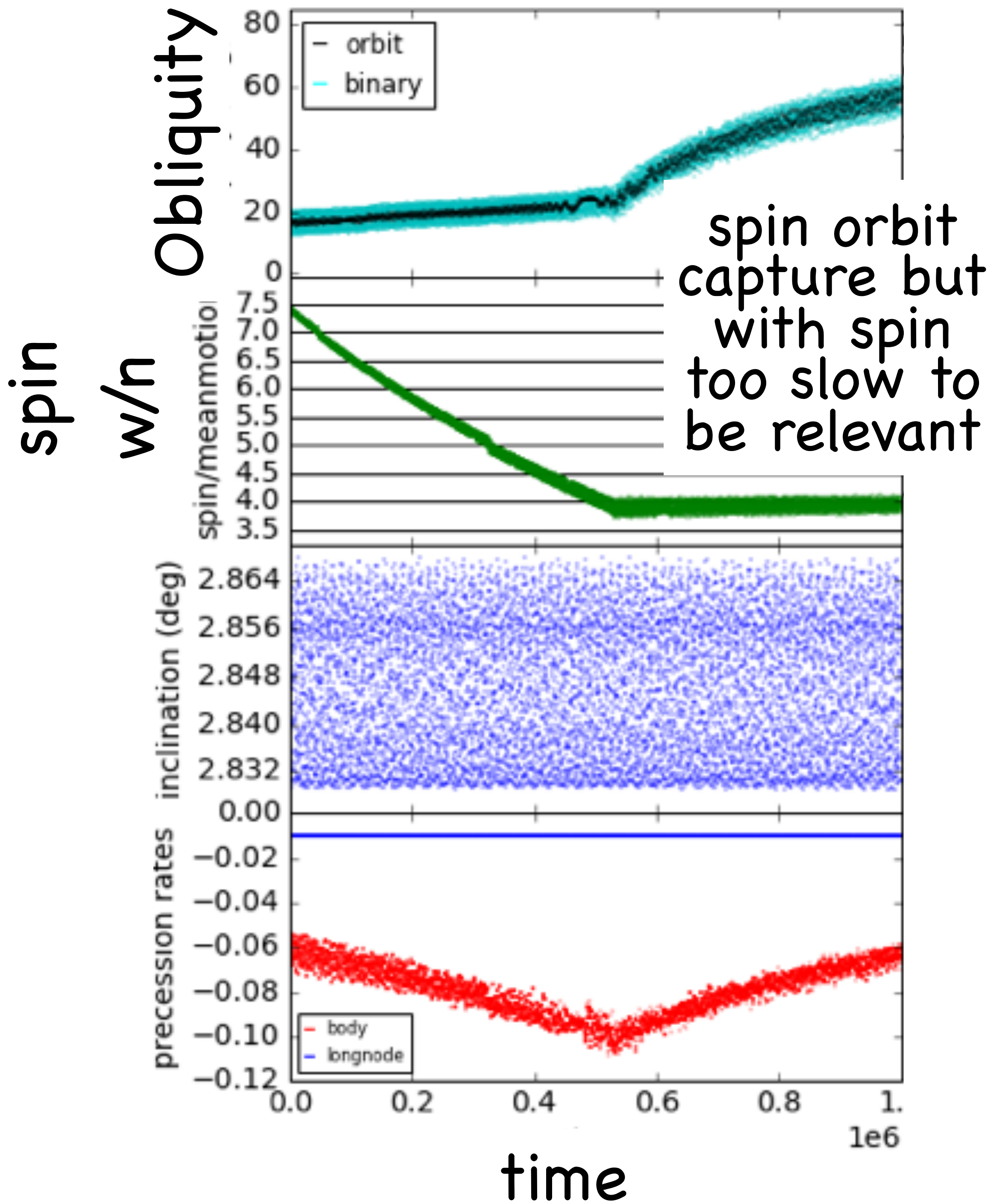
Tidal Evolution alone

Only Styx exhibits obliquity swings
Comparisons between simulations reveal that intermittent obliquity variations require binary, at current mass ratio, near 3:1 mean motion resonance.

How do we explain obliquities of **all** the satellites?

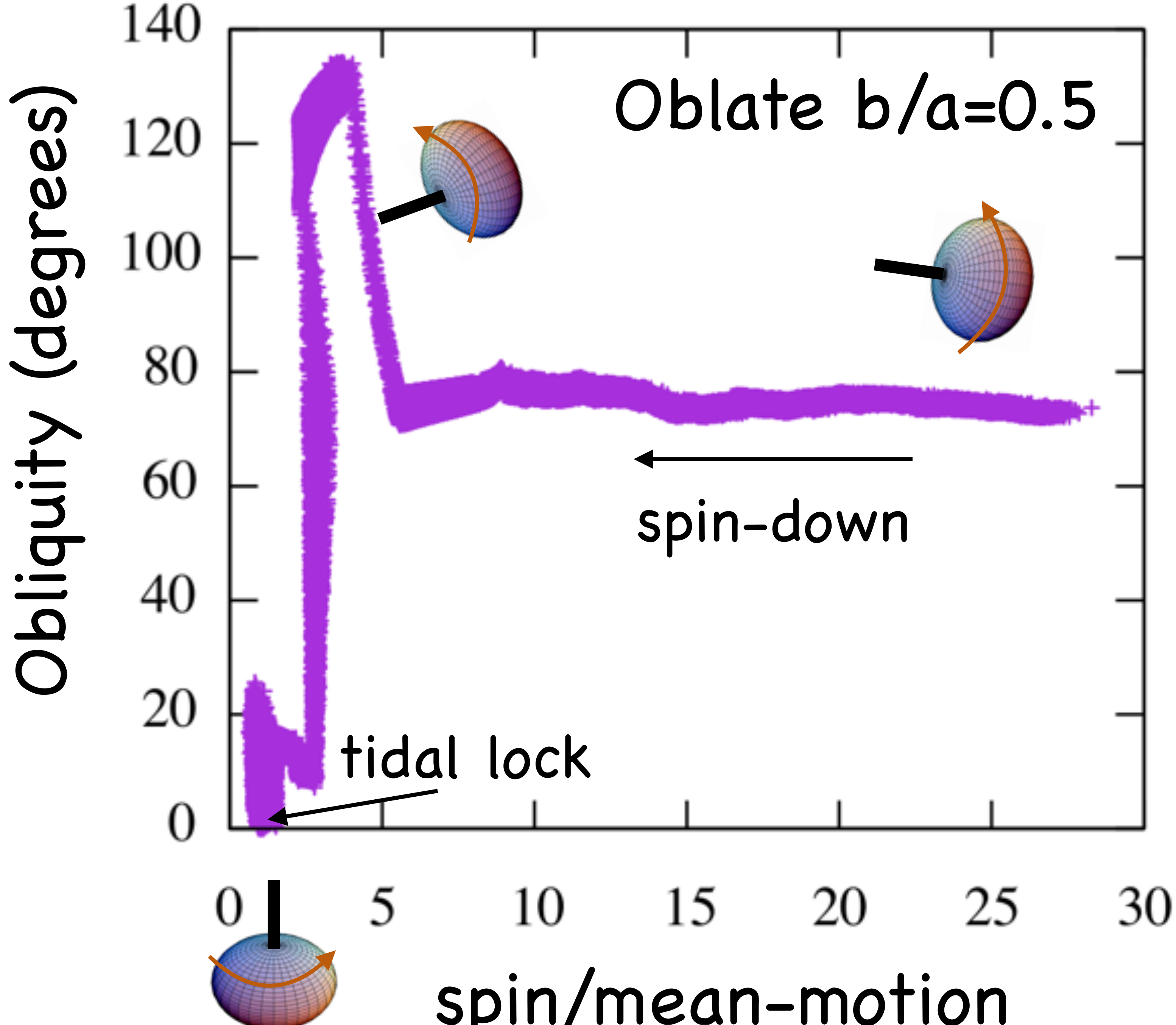
Maybe there was orbital migration due to a disk, not just due to tides

Circumbinary Obliquity evolution with some tidal evolution - Kerberos



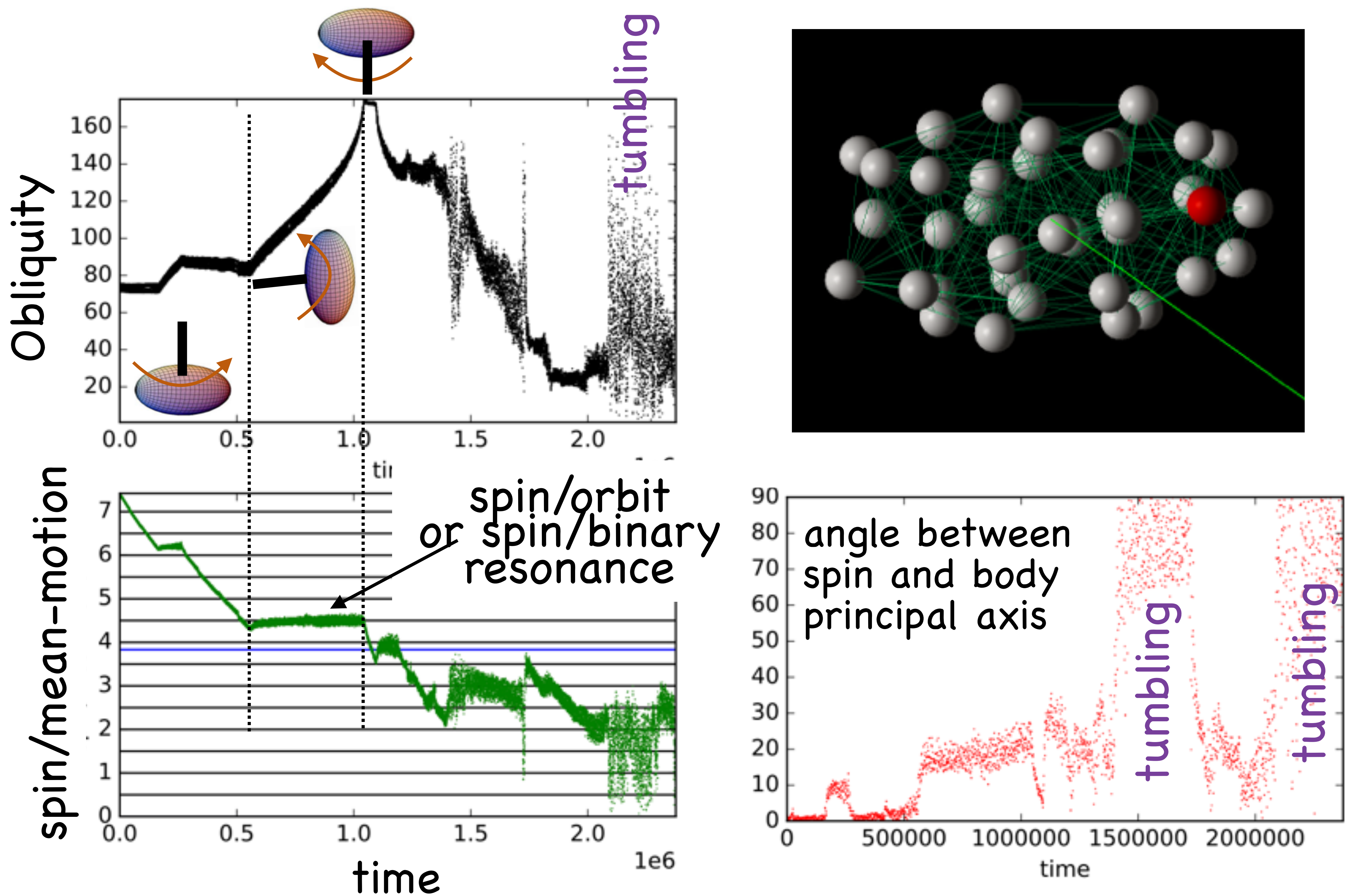
There are mechanisms for obliquity increase but they either don't work at current spin or are unlikely

Obliquity evolution (around a single mass)

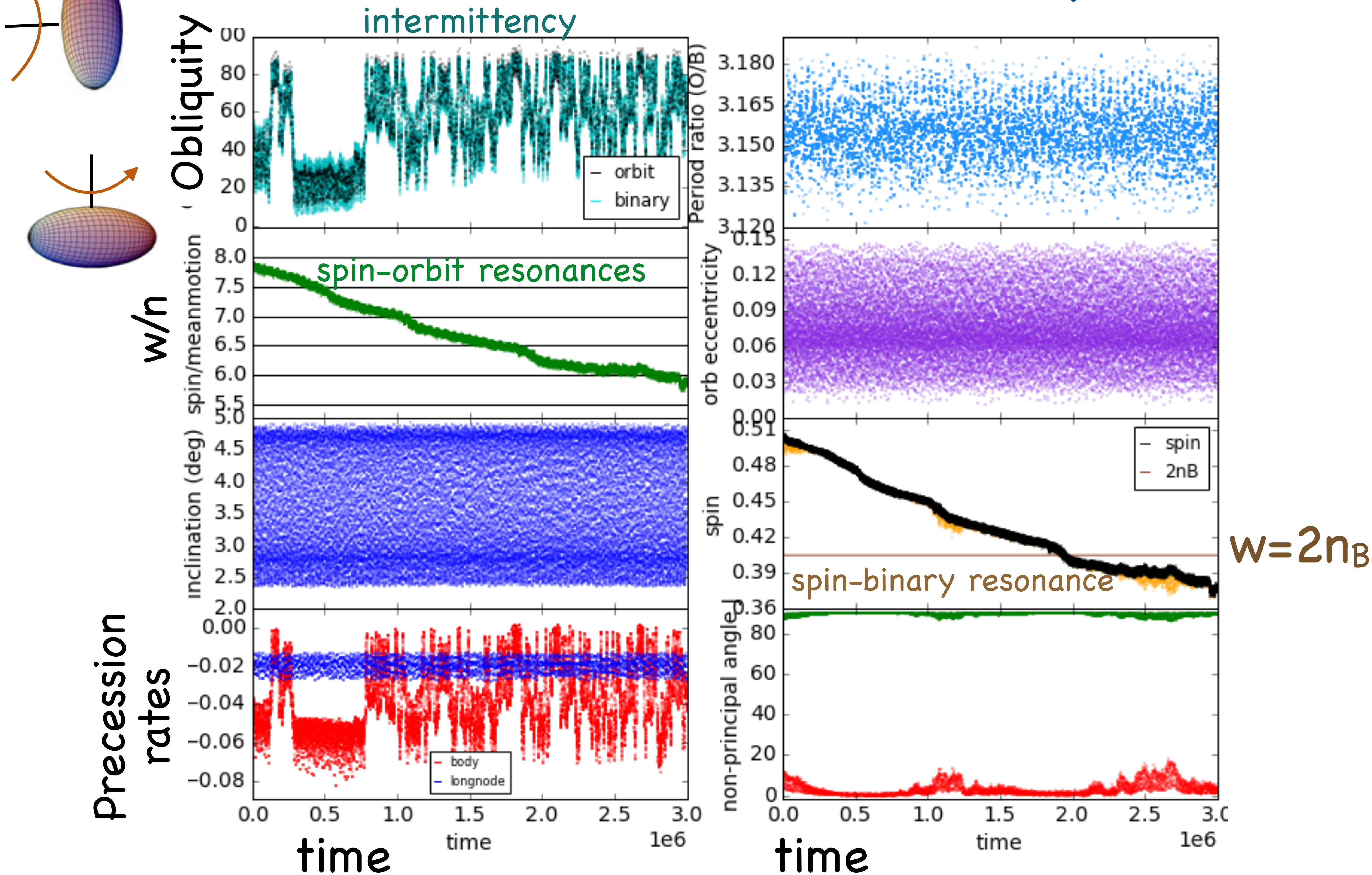


attitude instability in spin-orbit resonances

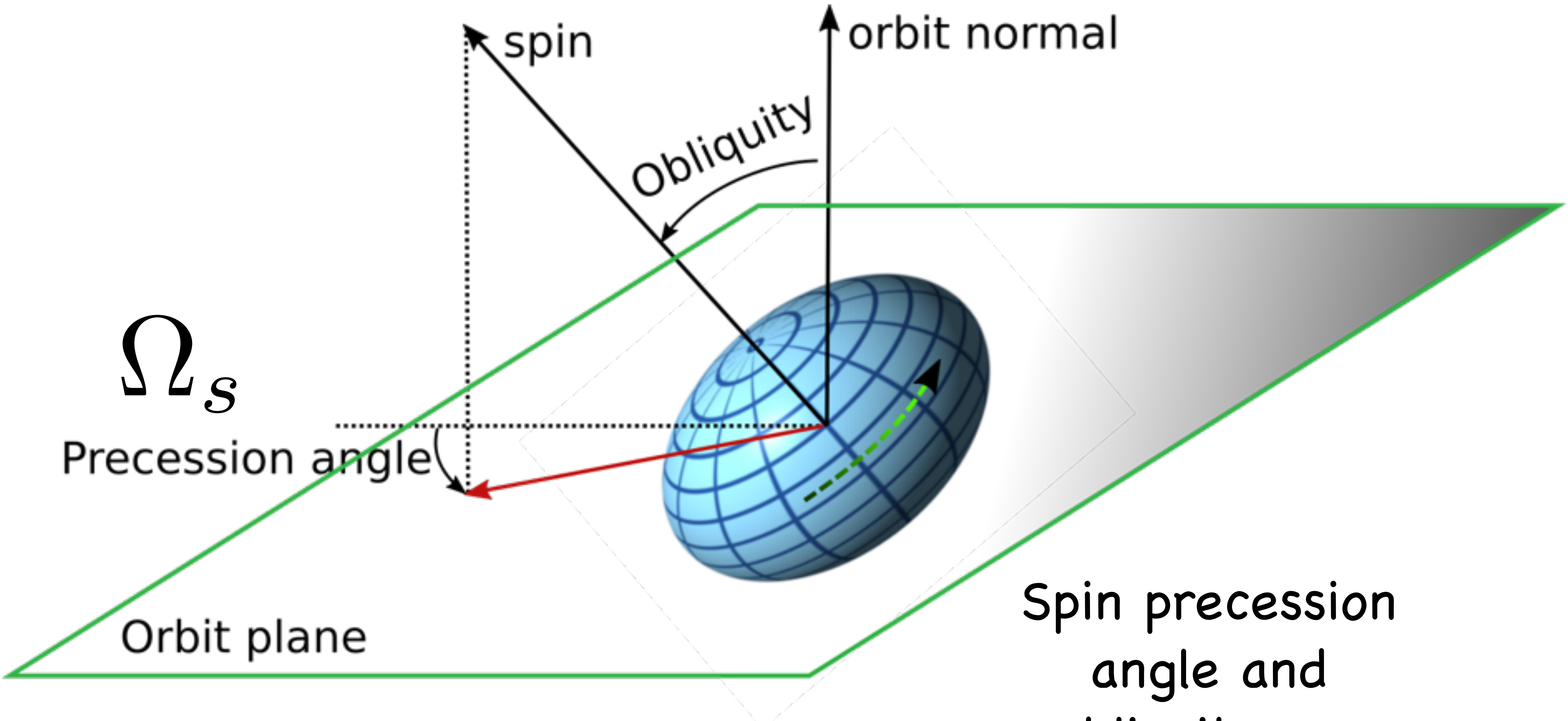
Circumbinary Obliquity evolution



Circumbinary Obliquity evolution with some tidal evolution - Styx

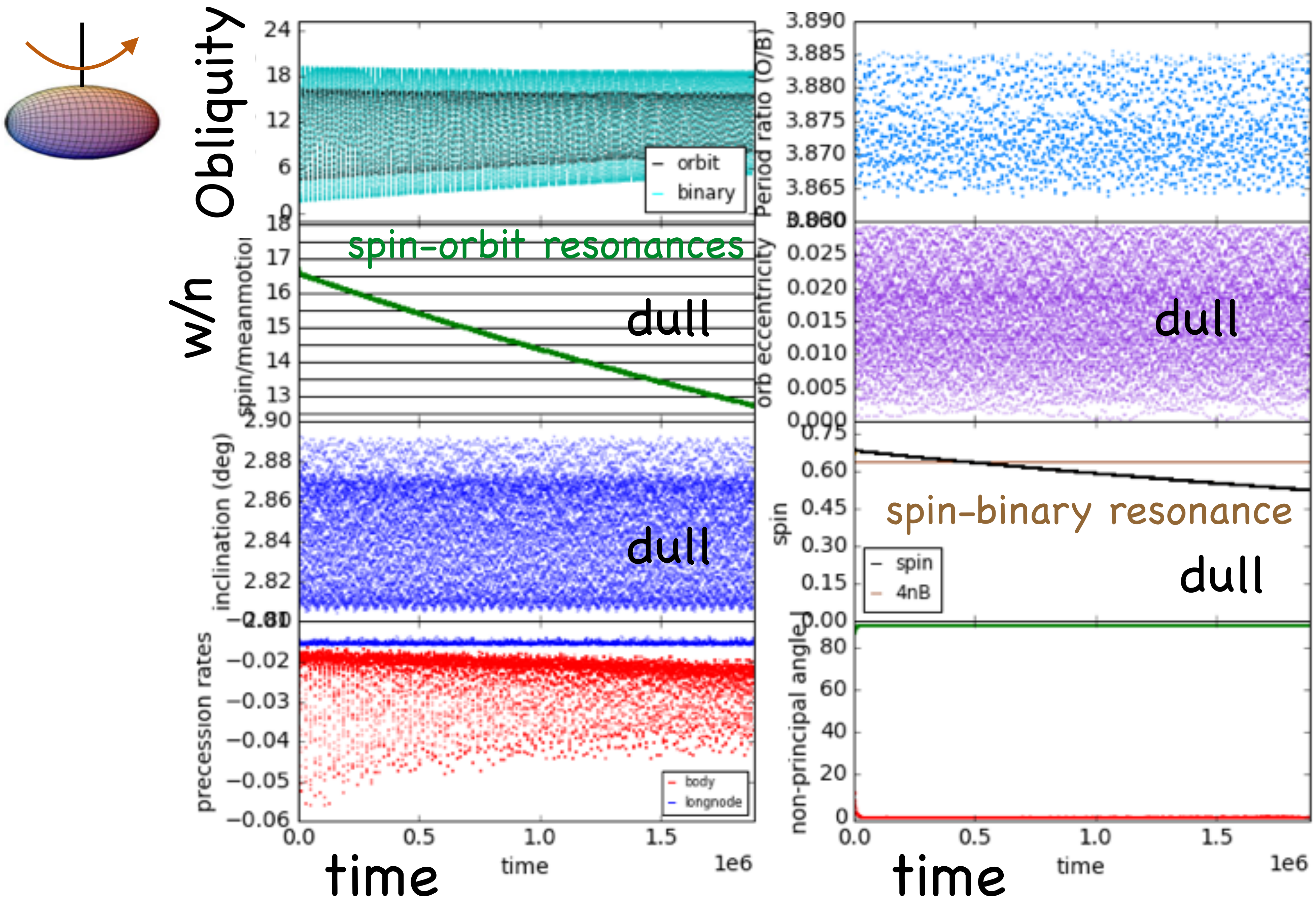


Spin Precession Angle



Spin precession angle and obliquity are Euler angles

Circumbinary Obliquity evolution with some tidal evolution - Nix



Nix is spinning so fast spin-orbit resonances are extremely weak

Spin-precession mean-motion resonance

retaining terms important near 3:1, 4:2, 5:3 .. (second order) mean motion resonances

$$H(p,\phi,\tau)^{j:j+2} = -\frac{p}{2}(2-p) + \beta c_0^j p(2-p)\cos(j\lambda-(j+2)\lambda'+2\phi) + (1-p)\sqrt{p(2-p)}\times \Big[\beta c_s^j s\sin(j\lambda-(j+2)\lambda'+\Omega+\phi) + \beta c_{s'}^j s'\sin(j\lambda-(j+2)\lambda'+\Omega'+\phi)\Big],$$

cos² obliquity

coefficients depend on Laplace coefs

sin cos obliquity

resonance strength depends on mass of perturber

These resonances are as strong as the secular resonances affecting obliquity of Mars, Saturn

Spin-precession mean-motion resonance

Unperturbed precession

Time in units of precession rate

Resonant argument (previously guessed!)

$$H(p,\phi,\tau)^{j:j+2} = -\frac{p}{2}(2-p)$$

Does not depend on inclination or eccentricity

$$+ \beta c_0^j p(2-p)\cos(j\lambda - (j+2)\lambda' + 2\phi)$$

$$+ (1-p)\sqrt{p(2-p)}\times$$

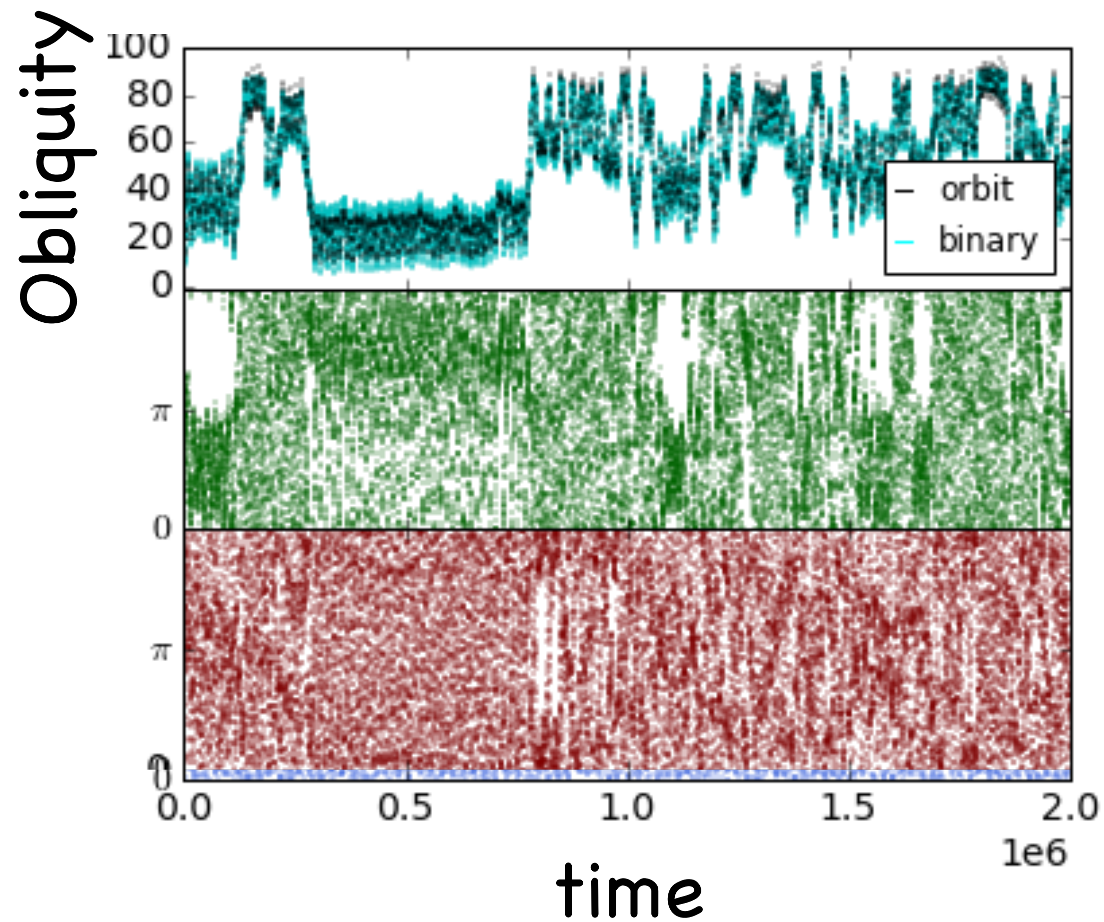
$$\Big[\beta c_s^j s \sin(j\lambda - (j+2)\lambda' + \Omega + \phi)$$

First order in inclination

$$+ \beta c_{s'}^j s' \sin(j\lambda - (j+2)\lambda' + \Omega' + \phi)\Big],$$

+ .... Additional terms ignored

Styx's intermittent Obliquity



Resonant Angles

$$3\lambda - \lambda_B - \Omega - \Omega_s$$

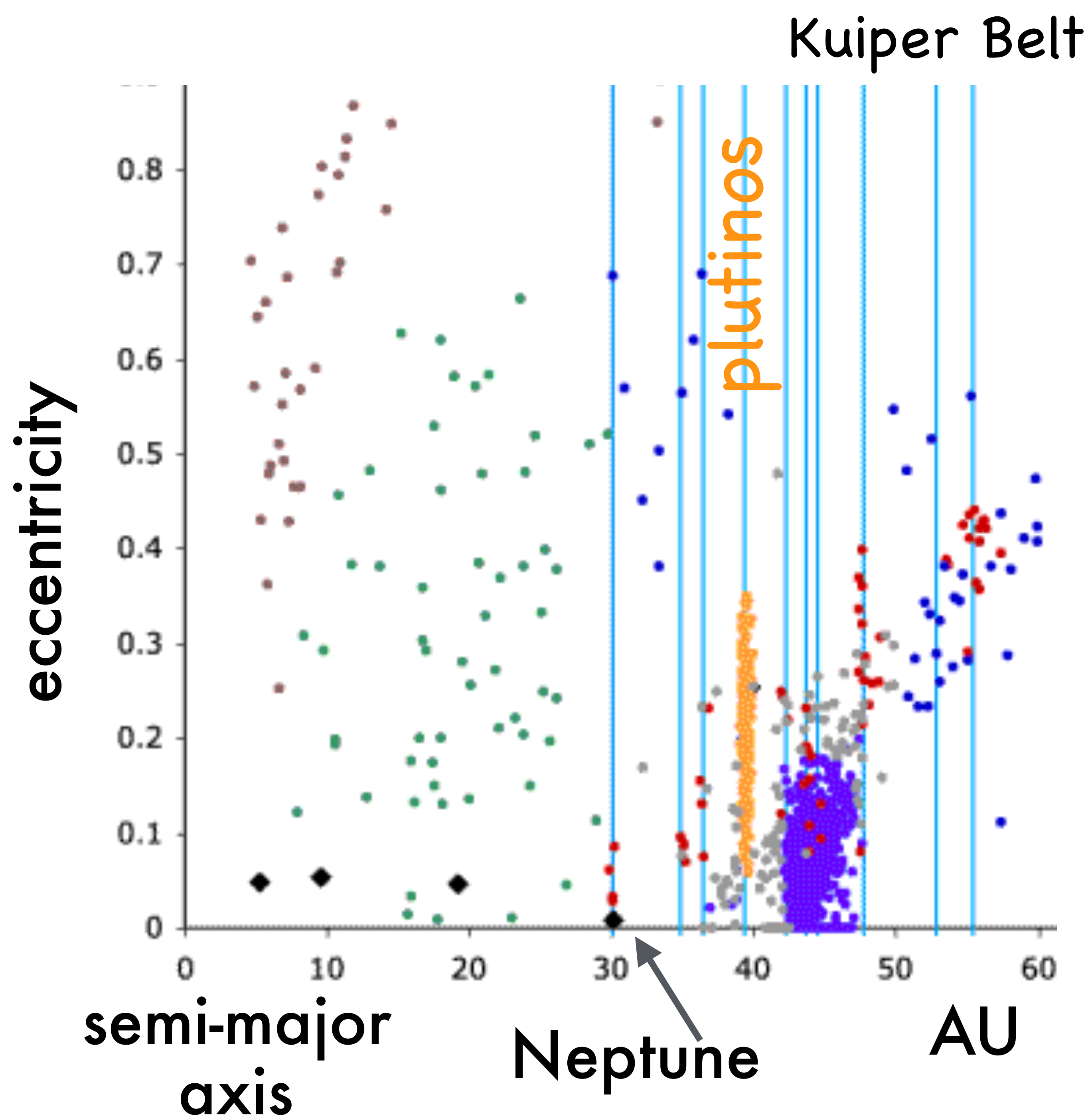
$$3\lambda - \lambda_B - 2\Omega_s$$

Styx tidal evolution alone

This resonance is responsible for the intermittency

Spin precession/mean motion resonance could be affecting Styx now

Why the effort to make a Hamiltonian model?



Resonance Capture model for Plutinos (Malhotra)

As Neptune moves outwards, Pluto's eccentricity increases