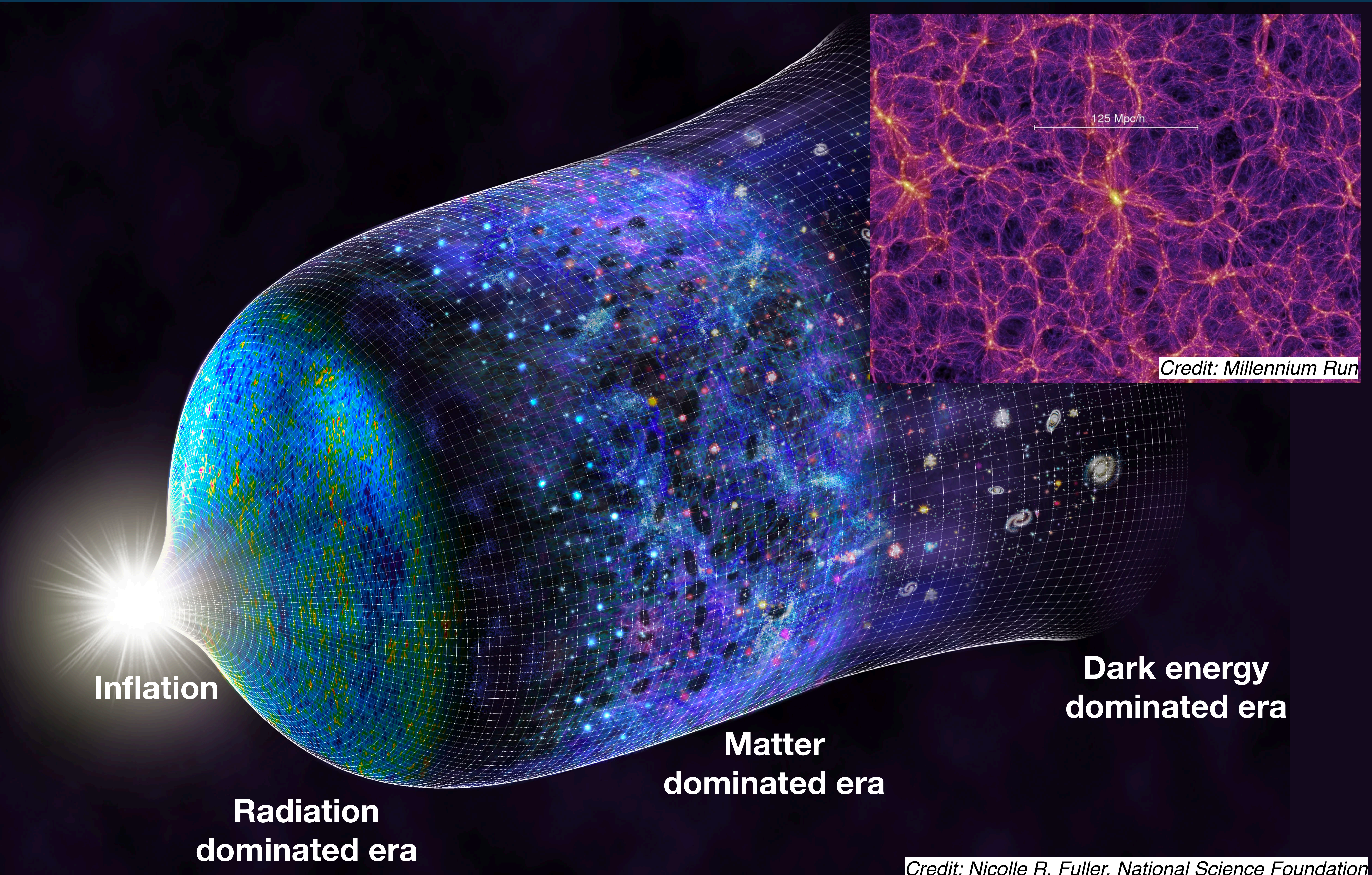
Characterising spacetime during cosmological structure formation

Robyn L. Munoz
Supervisor: **Dr. Marco Bruni**

Introduction

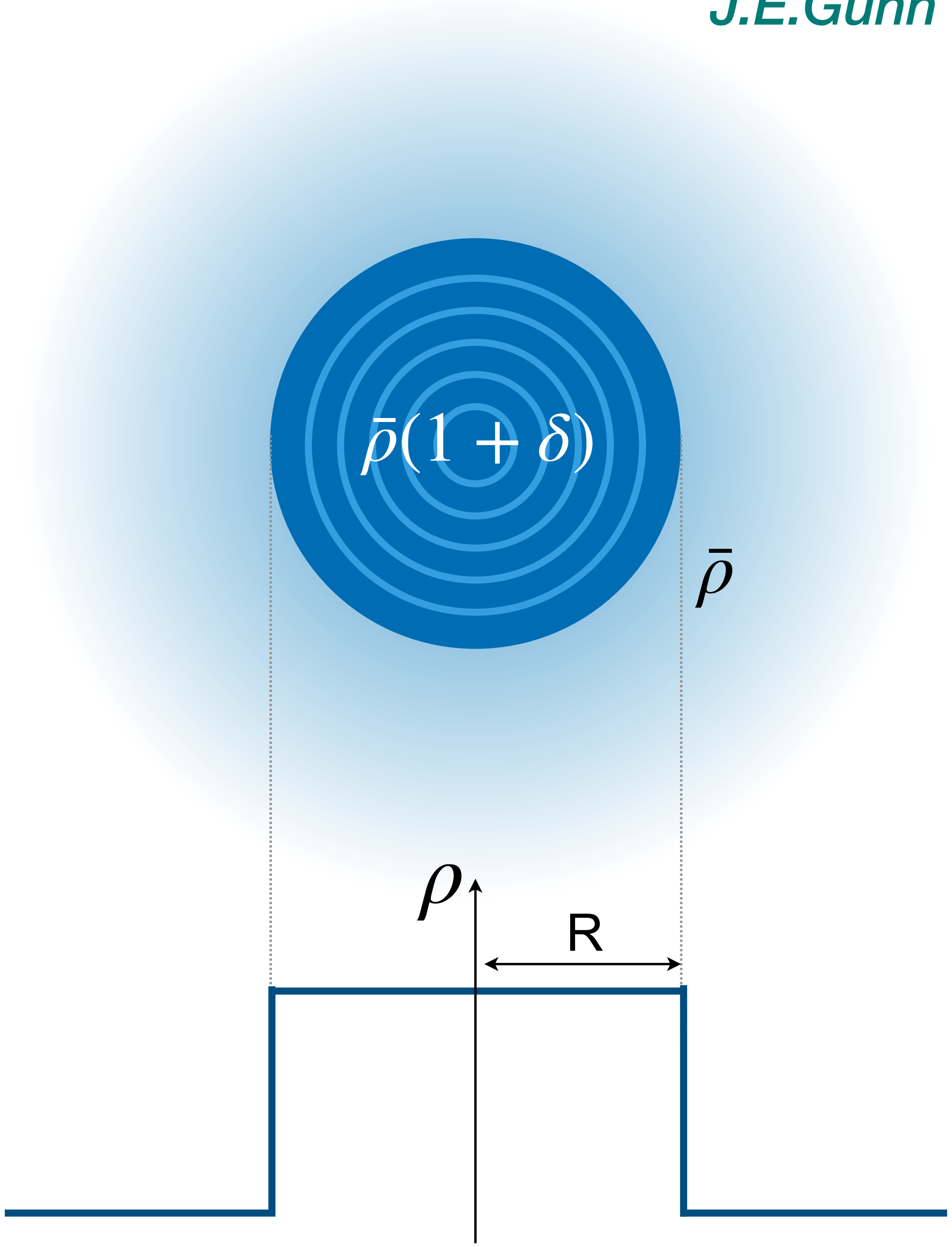


Credit: Millennium Run

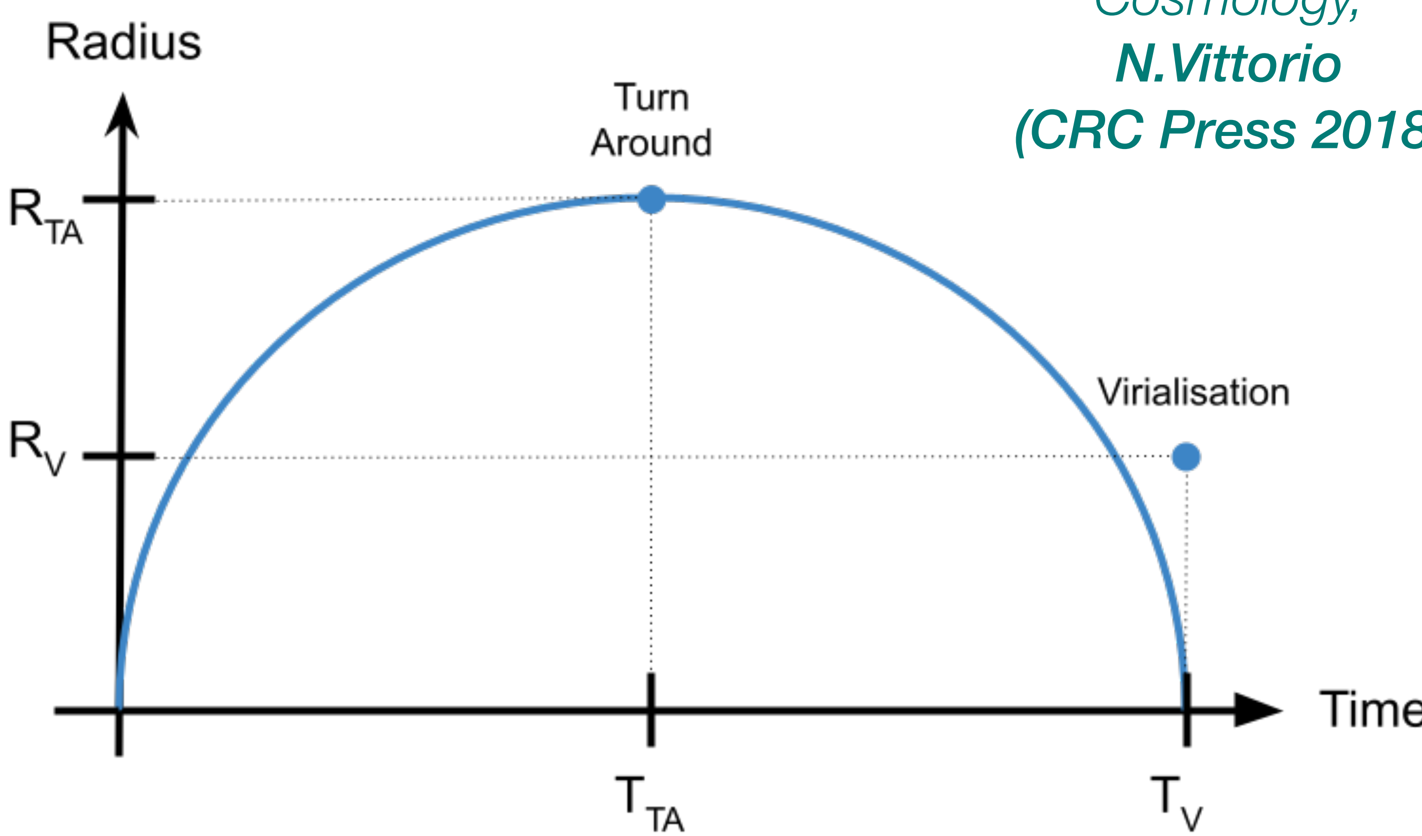
Credit: Nicolle R. Fuller, National Science Foundation

Top-Hat Spherical Collapse Model

On the infall of matter into clusters of galaxies and some effects on their evolution,
J.E.Gunn and J.R.Gott (ApJ. 1972)



Cosmology,
N.Vittorio
(CRC Press 2018)



$$\delta_c^{(1)} = 1.68$$
$$\delta_c = 177$$

Important for Press-Schechter mass function

Introduction

Objective: **Study the growth of large scale structures with simulations in numerical relativity.**

- Implement initial conditions of an inhomogeneous ΛCDM universe, from a fully growing mode defined from the curvature perturbation $\mathscr{R}_c$.
- Explore the validity of the top-hat spherical collapse model.
- Gravito-electromagnetic characterisation of this simulation.
- Classify the spacetime according to its Petrov type.

Introduction

Objective: **Study the growth of large scale structures with simulations in numerical relativity.**

arXiv > astro-ph > arXiv:2302.09033

Astrophysics > Cosmology and Nongalactic Astrophysics

[Submitted on 17 Feb 2023]

Structure formation and quasi-spherical collapse from initial curvature perturbations with numerical relativity simulations

Robyn L. Munoz, Marco Bruni

arXiv > gr-qc > arXiv:2211.08133 robynlm

General Relativity and Quantum Cosmology

[Submitted on 15 Nov 2022]

EBWeyl: a Code to Invariantly Characterize Numerical Spacetimes

Robyn L. Munoz, Marco Bruni

Initial Conditions

Background:

- Flat FLRW metric,
- ΛCDM with pressureless perfect fluid,
- Matter-dominated era.

- $\mathscr{R}_c$ and $\zeta^{(1)}$ are used to quantify perturbations created during inflation
- $\mathscr{R}_c$ is gauge invariant at first order
- $\dot{\mathscr{R}}_c = 0$

Inhomogeneity:

- Synchronous and comoving gauge
- Scalar perturbations,

Non-Gaussian initial conditions in ΛCDM: Newtonian, relativistic, and primordial contributions, ***M.Bruni, J.C.Hidalgo, N.Meures and D.Wands (ApJ. 2014)***

Comoving Curvature Perturbation: $\mathscr{R}_c$ → γ_{ij} → K_{ij} and ${}^{(3)}R$ → ρ

Perturbed up to 1st order

From the Hamiltonian constraint equation

γ_{ij}: Spatial metric

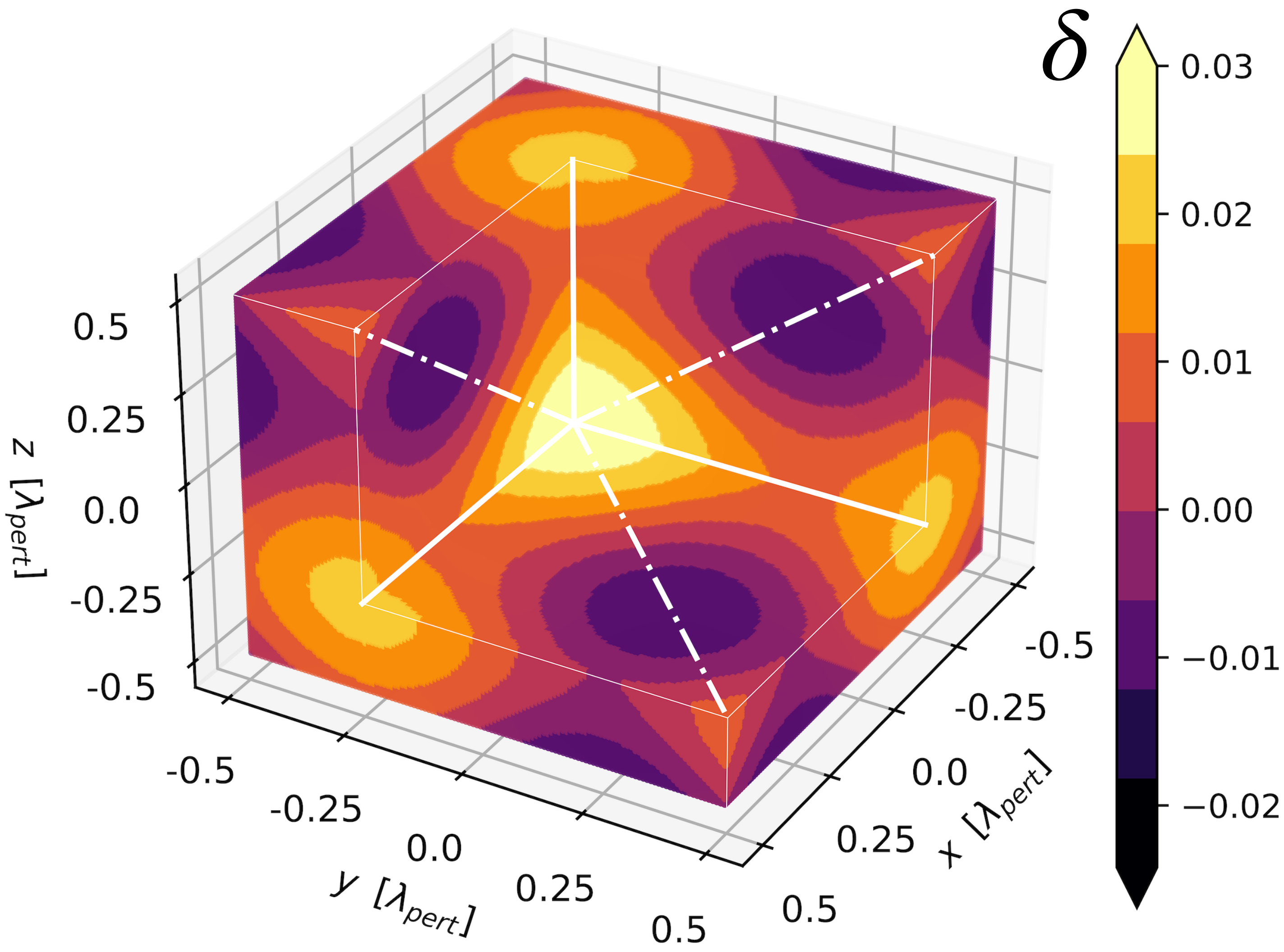
K_{ij}: Extrinsic curvature

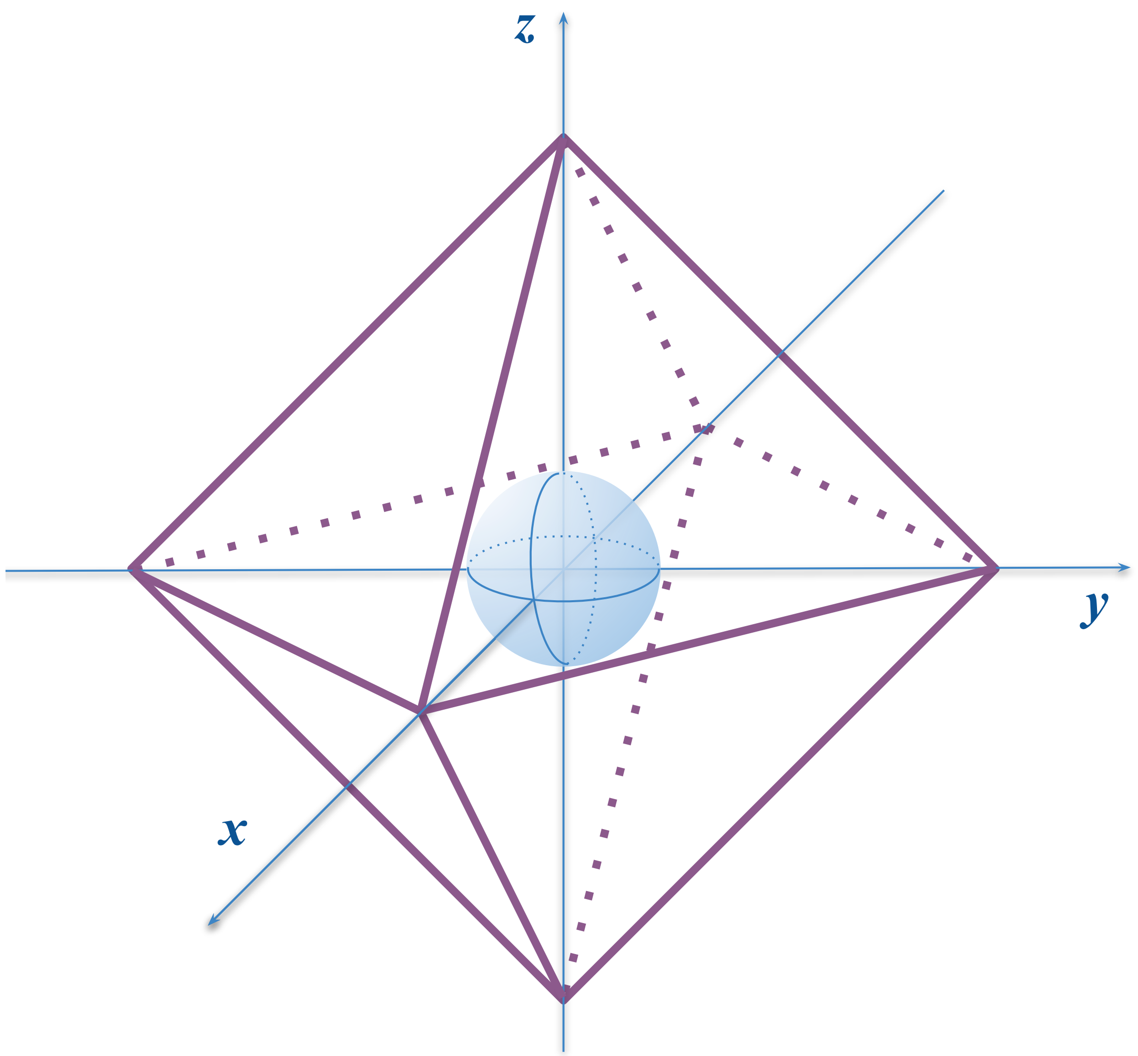
${}^{(3)}R$: 3 Ricci scalar

ρ: Energy density

Initial Conditions

$$\mathscr{R}_c = A_{pert}\left[\sin\left(\frac{2\pi x}{\lambda_{pert}}\right) + \sin\left(\frac{2\pi y}{\lambda_{pert}}\right) + \sin\left(\frac{2\pi z}{\lambda_{pert}}\right)\right]$$





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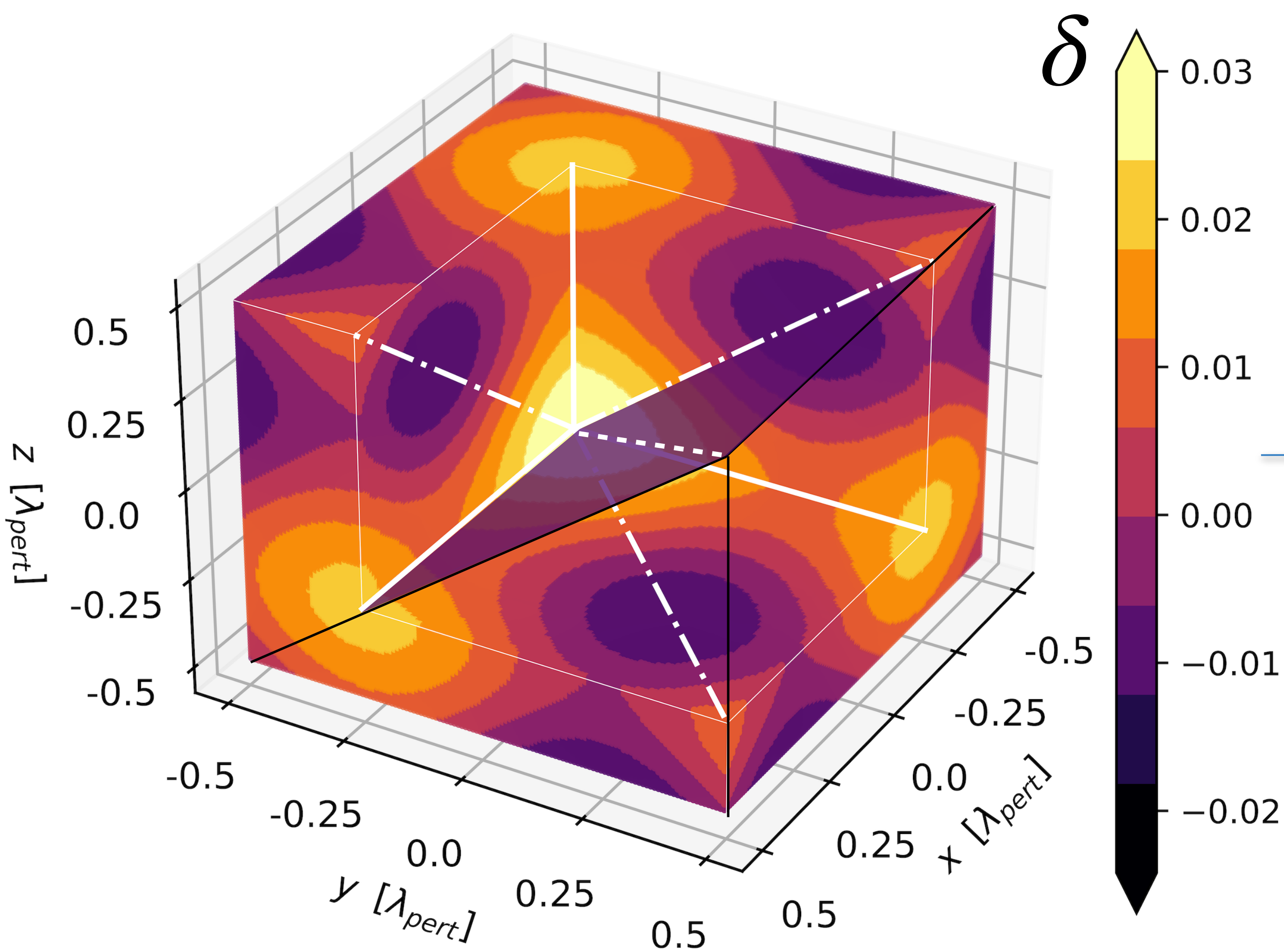

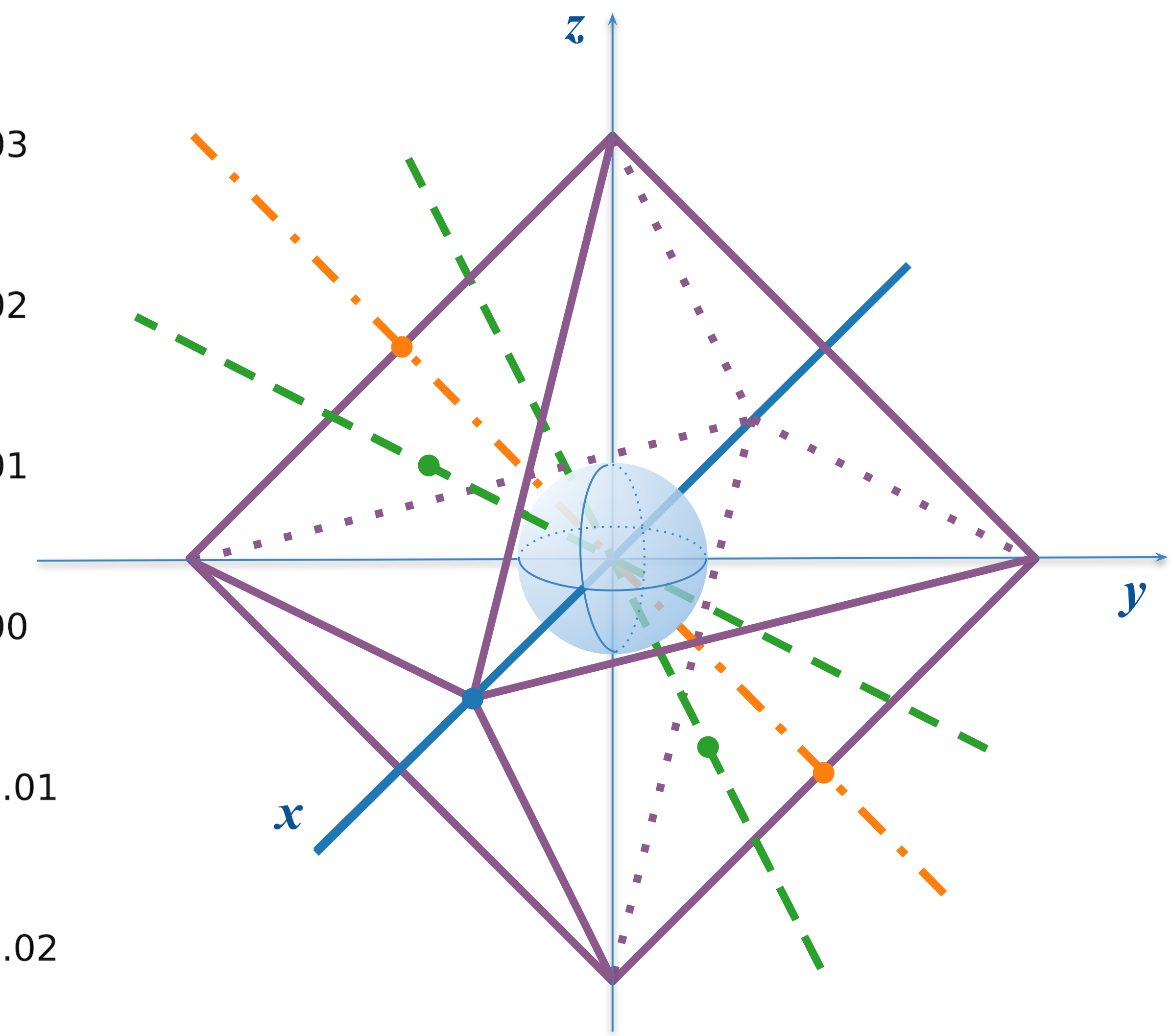

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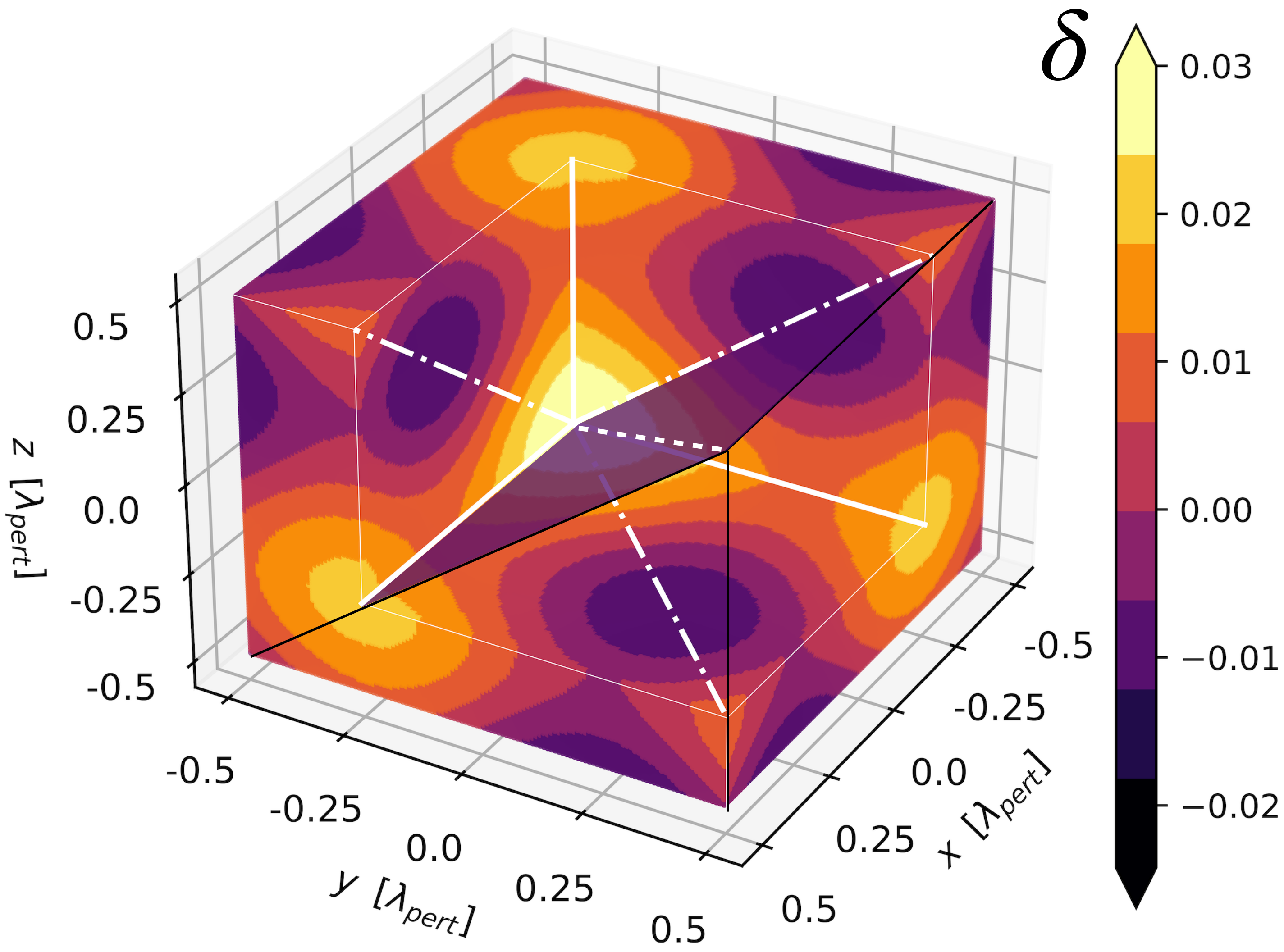
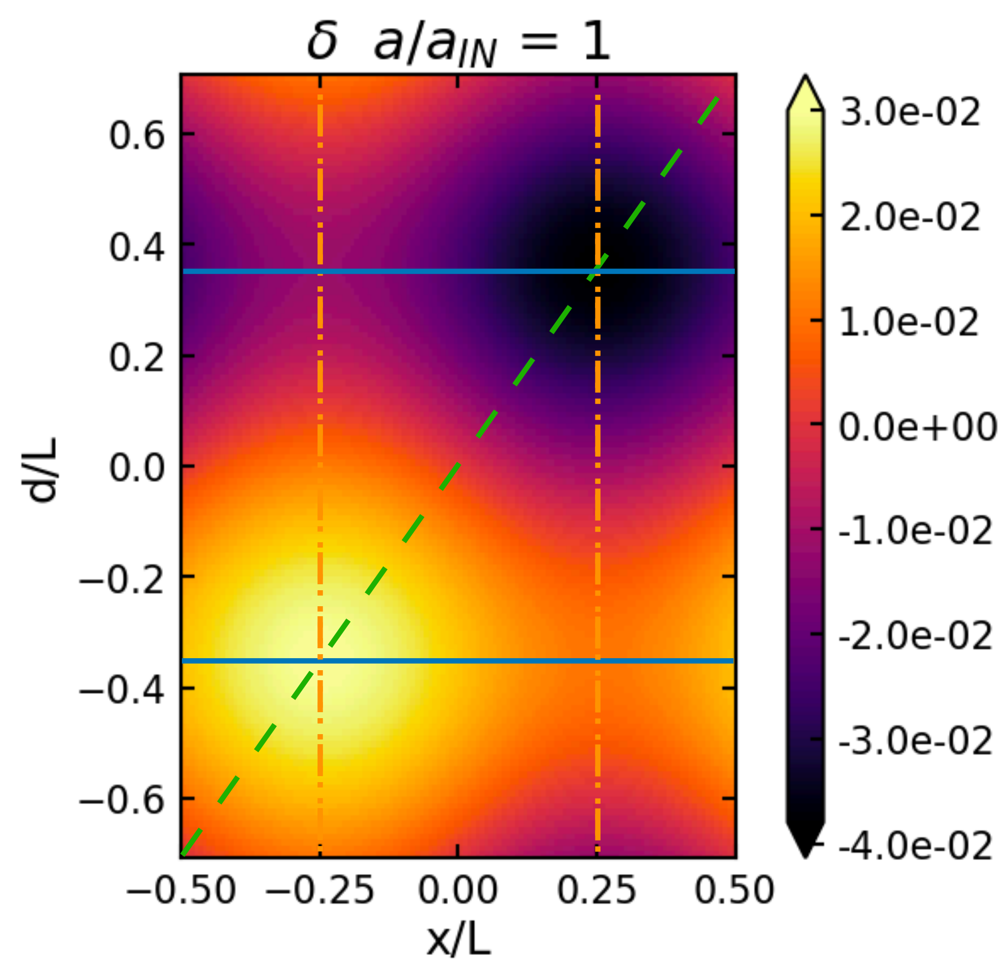


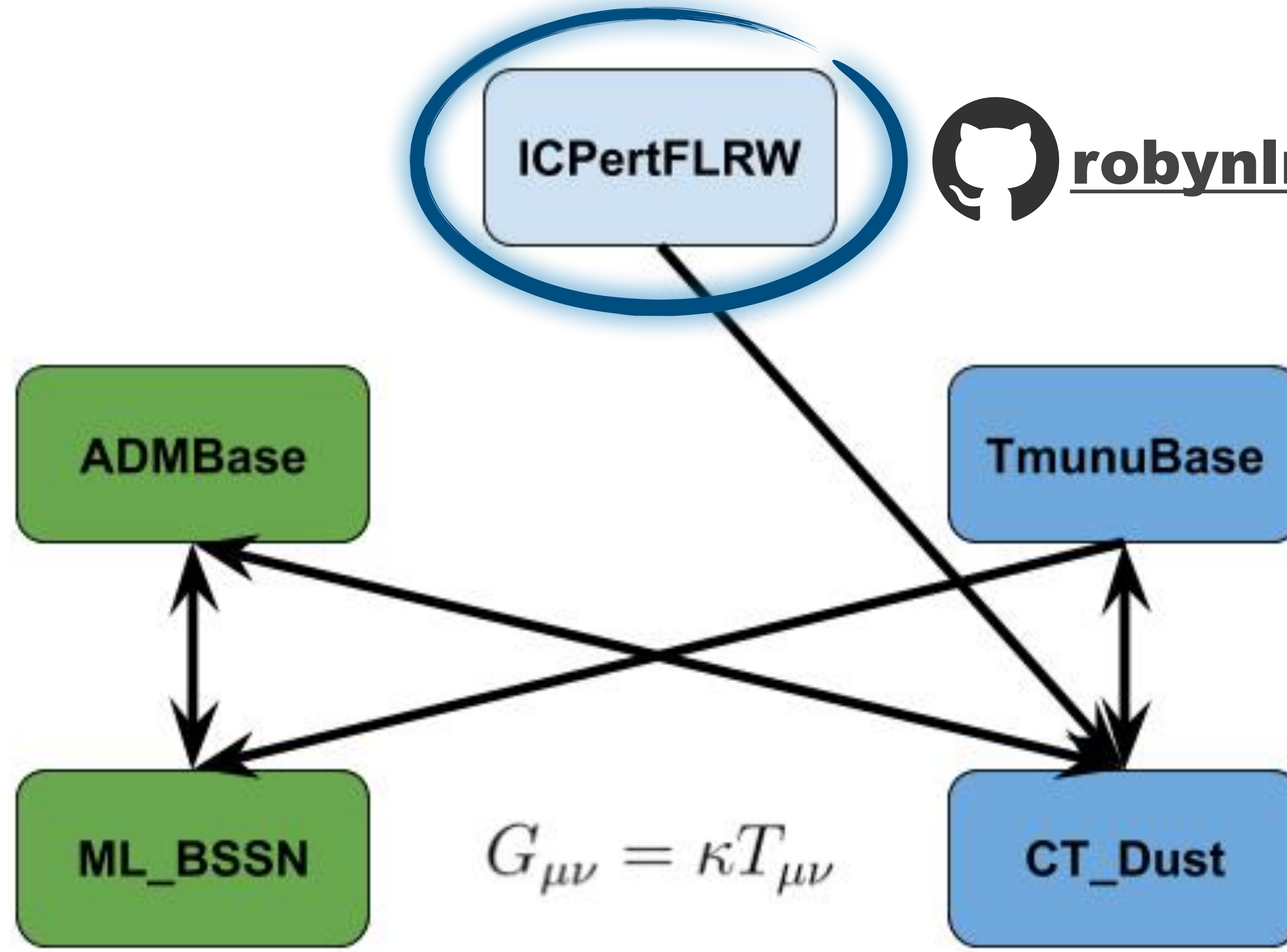


Initial Conditions

I wrote a Fortran thorn to implement these initial conditions in Einstein Toolkit

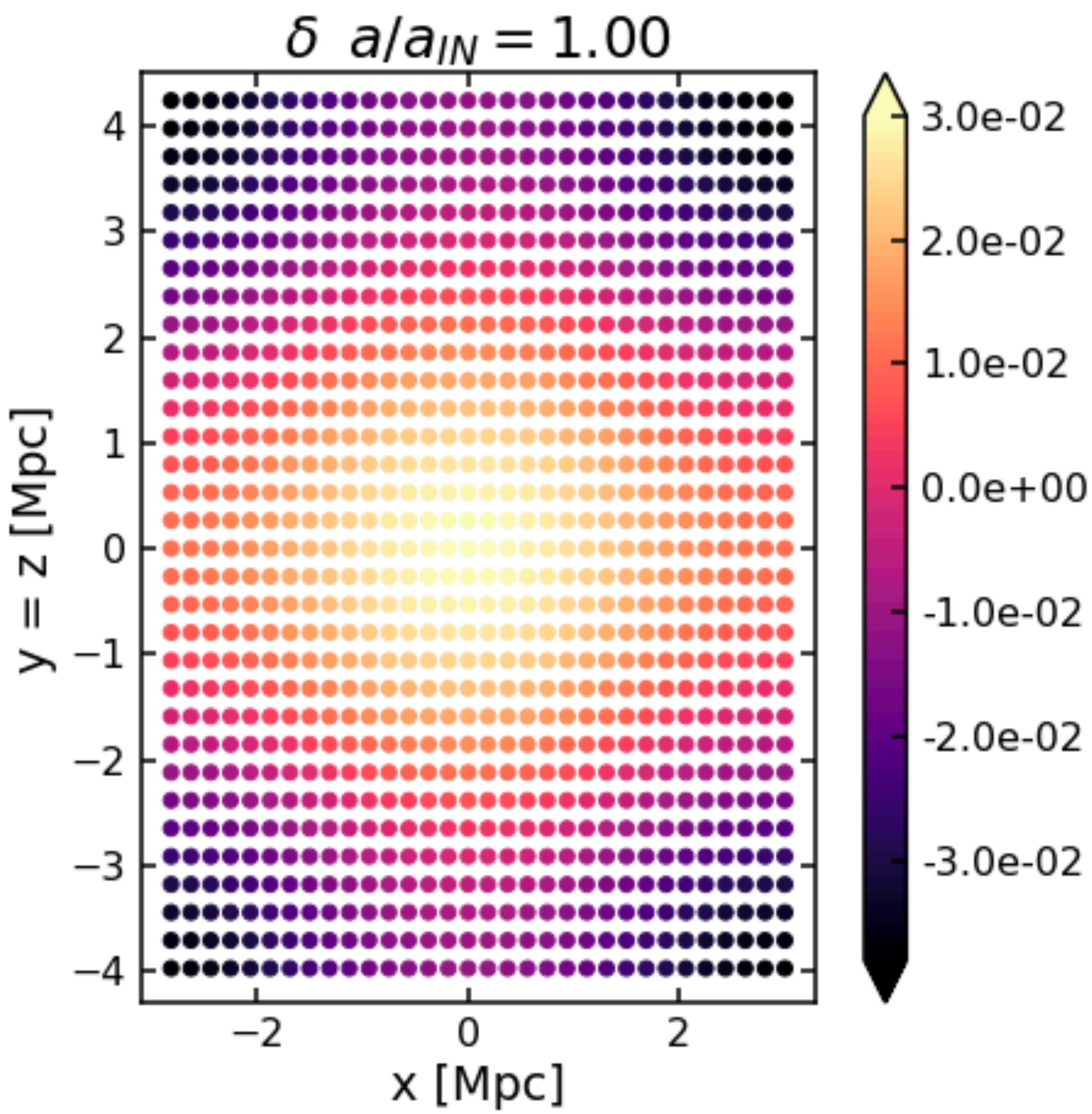
einstein toolkit

robynlm



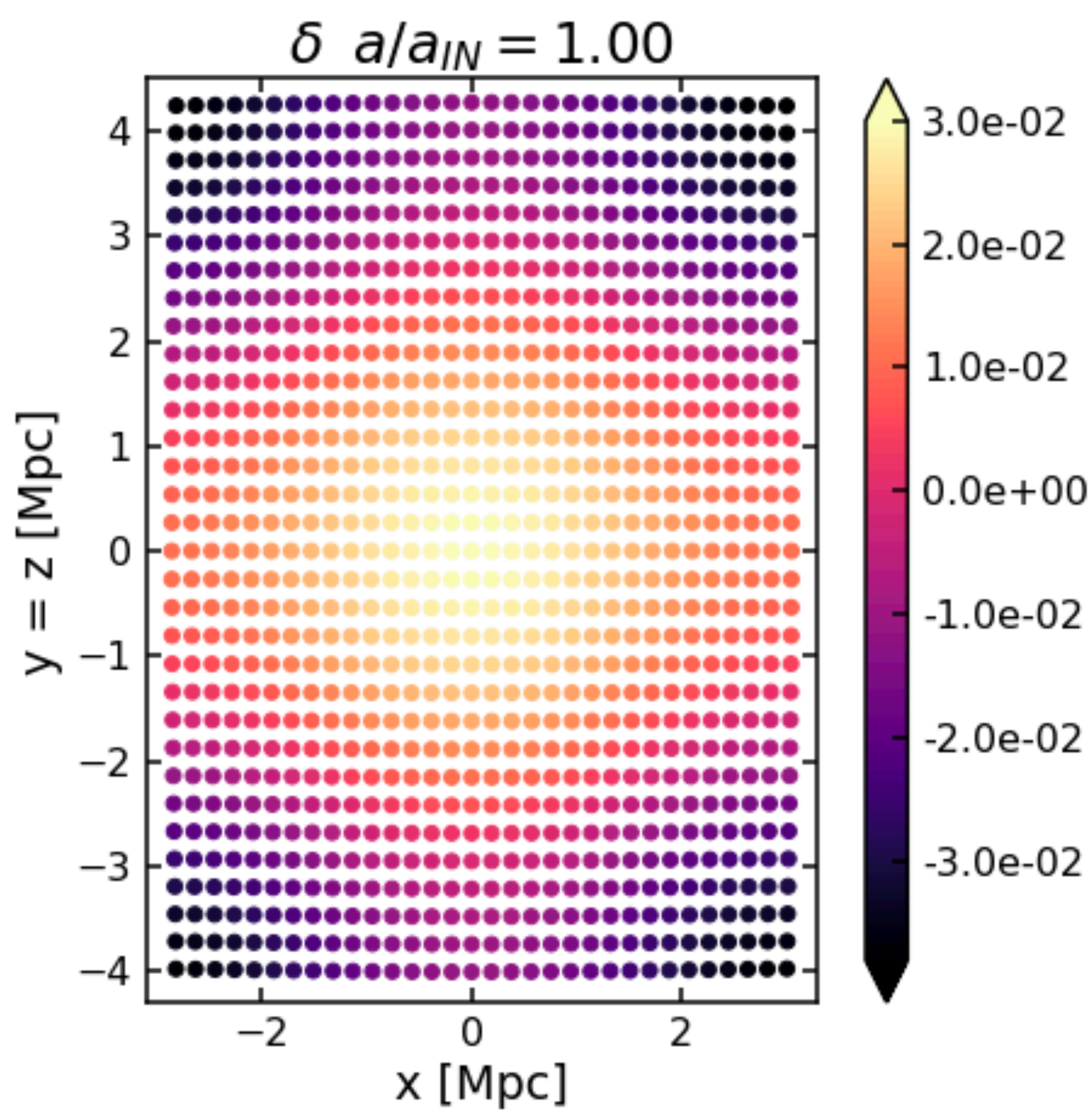
Collapse evolution

Evolution of each position in the simulation box



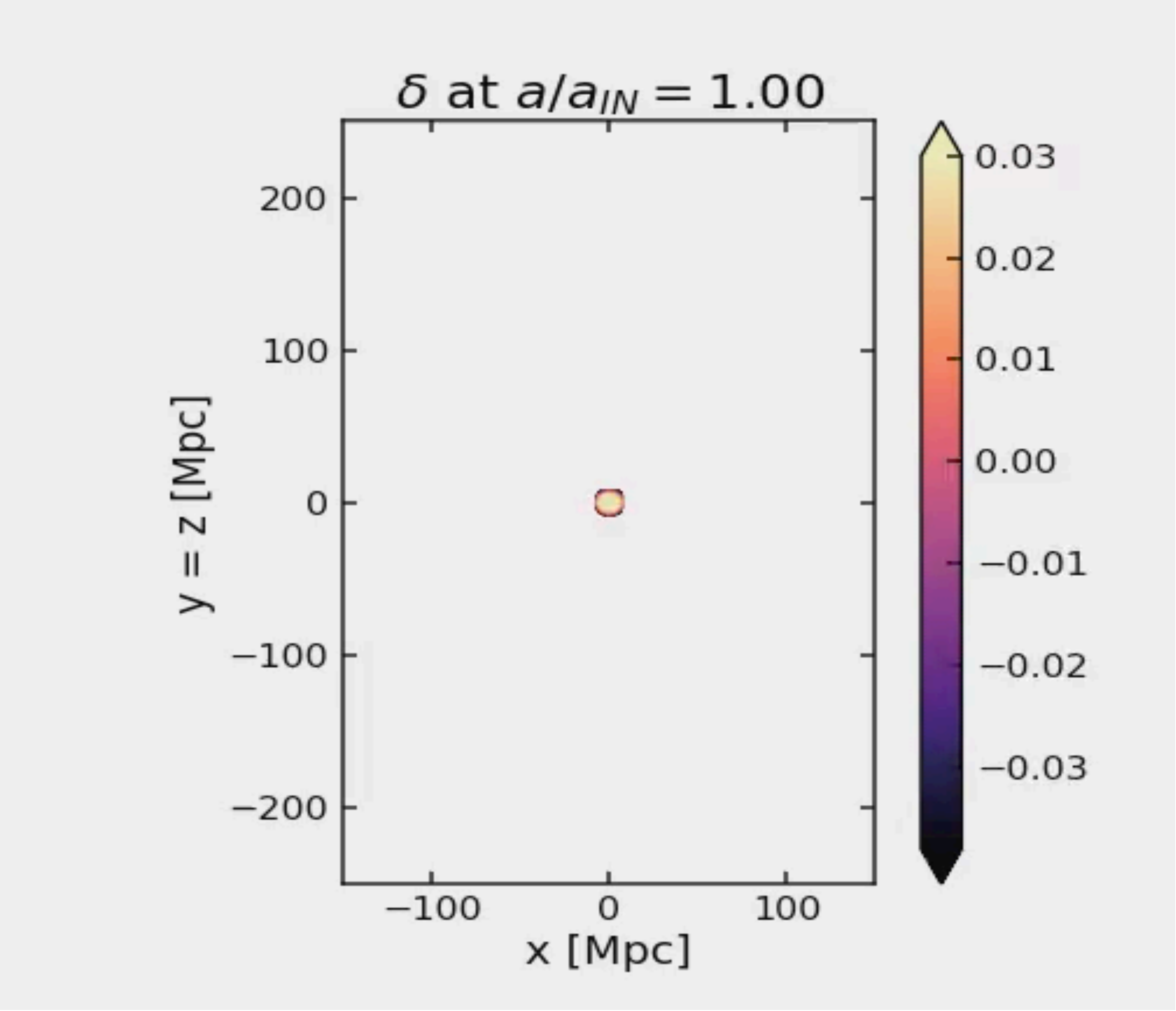
Collapse evolution

Evolution of each position in the simulation box in **proper length** with respect to the centre of the over-density



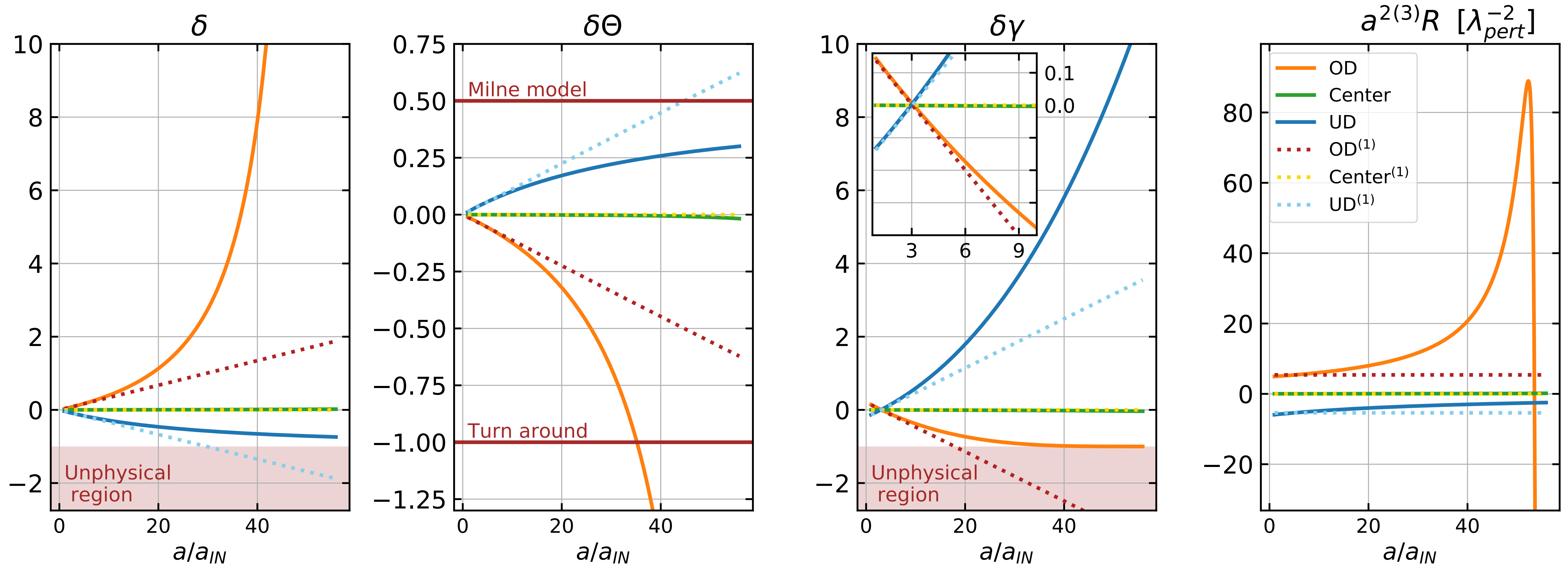
Collapse evolution

Evolution of each position in the simulation box in <u>proper length</u> with respect to the centre of the over-density



Collapse evolution

$\delta_{OD,\,IN} = 3 \times 10^{-2}$ $\lambda_{pert} = 1821$Mpc $z_{IN} = 302.5$



Perturbation in:

Matter density:

$\delta = \rho/\bar{\rho} - 1$

Trace of expansion:

$\delta\Theta = \Theta/\bar{\Theta} - 1$

Determinant of spatial metric:

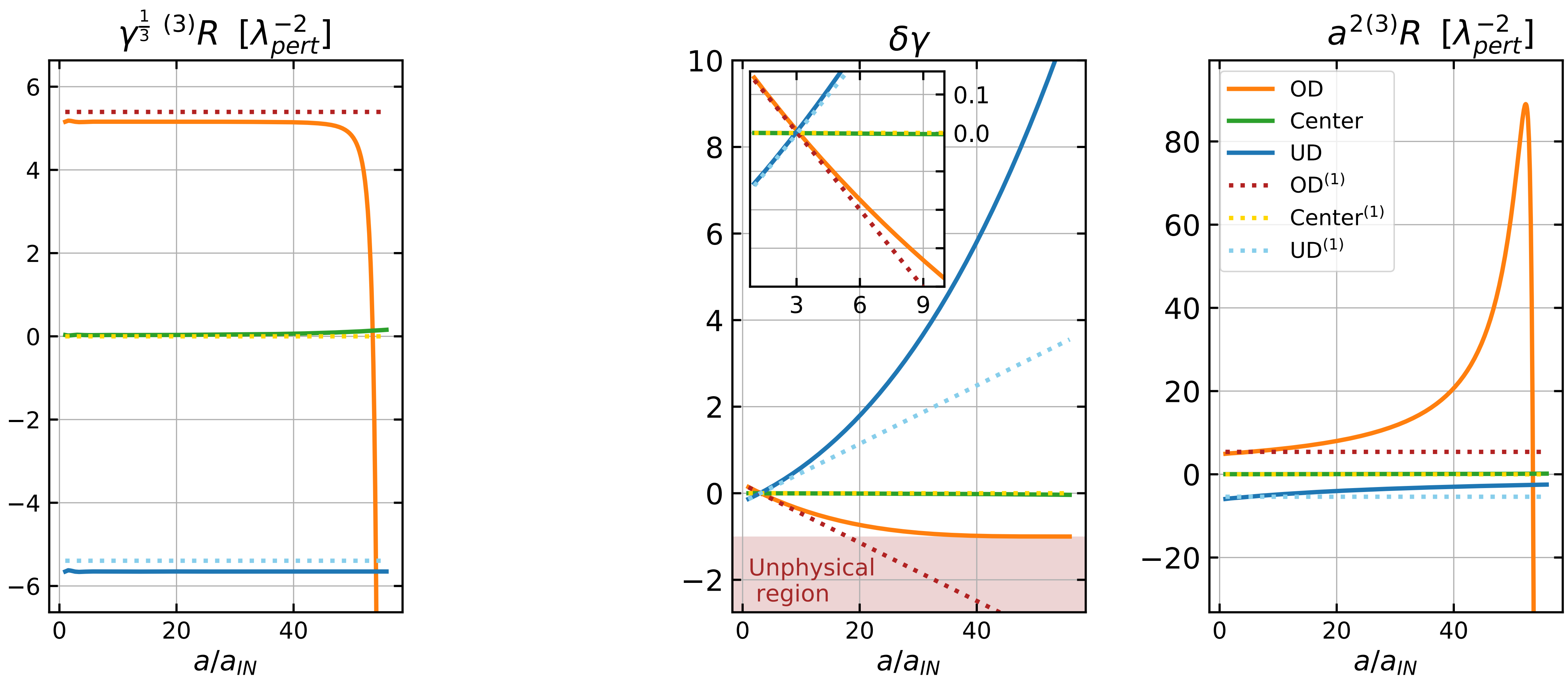
$\delta\gamma = \gamma/\bar{\gamma} - 1$

Conformal 3 Ricci scalar:

$a^{2(3)}R$

Collapse evolution

$\delta_{OD,\,IN} = 3 \times 10^{-2} \quad \lambda_{pert} = 1821\text{Mpc} \quad z_{IN} = 302.5$



Local conformal 3 Ricci scalar:
$\gamma^{\frac{1}{3}}\ {}^{(3)}R$

Perturbation in:

Determinant of spatial metric:

$\delta\gamma = \gamma/\bar{\gamma} - 1$

Conformal 3 Ricci scalar:

$a^{2(3)}R$

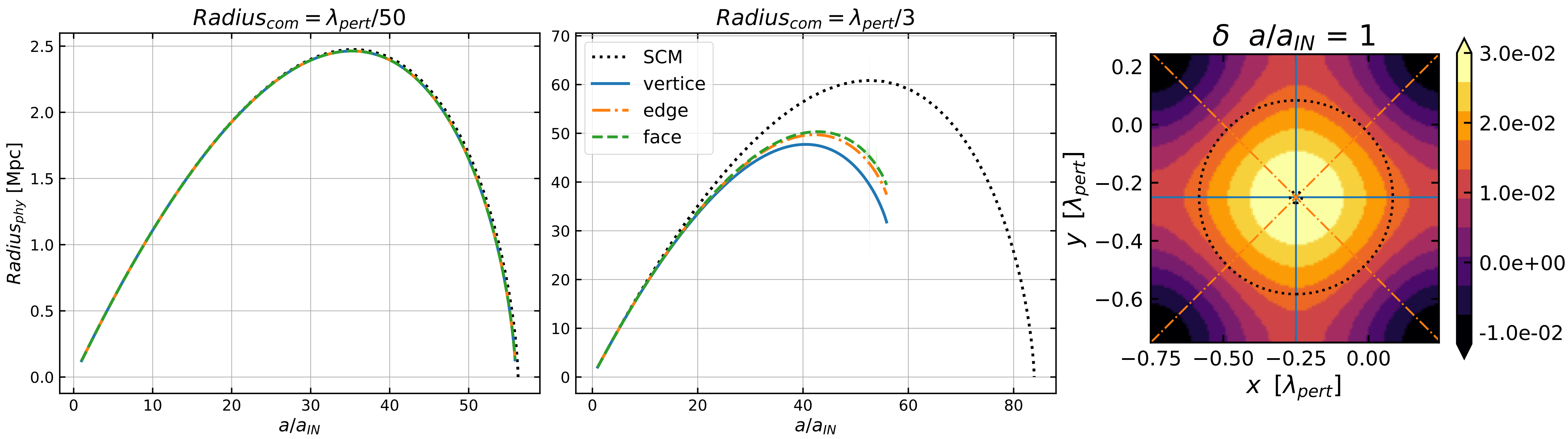
Collapse evolution

$\delta_{OD,\, IN} = 3.10^{-2}$ $\delta^{(1)} = \delta_{IN}\ a/a_{IN}$		Top Hat	Here
Turn Around $\Theta = 0$	a/a_{IN}	35.3	35.195 ± 3e-3
	$\delta^{(1)}_{OD}$	1.06	1.05584 ± 8e-5
	δ_{OD}	4.55	4.5626 ± 5e-4
Virialisation / Crash	a/a_{IN}	56	55.87 ± 8e-2
	$\delta^{(1)}_{OD}$	1.68	1.676 ± 2e-3
	δ_{OD}	177	

At the peak of the over-density the top-hat spherical collapse model is an excellent approximation.

Collapse evolution

The proper radius of a comoving sphere centred on the over-density compared to the spherical collapse model with $\delta = \langle \delta \rangle_{\mathscr{D}}$.

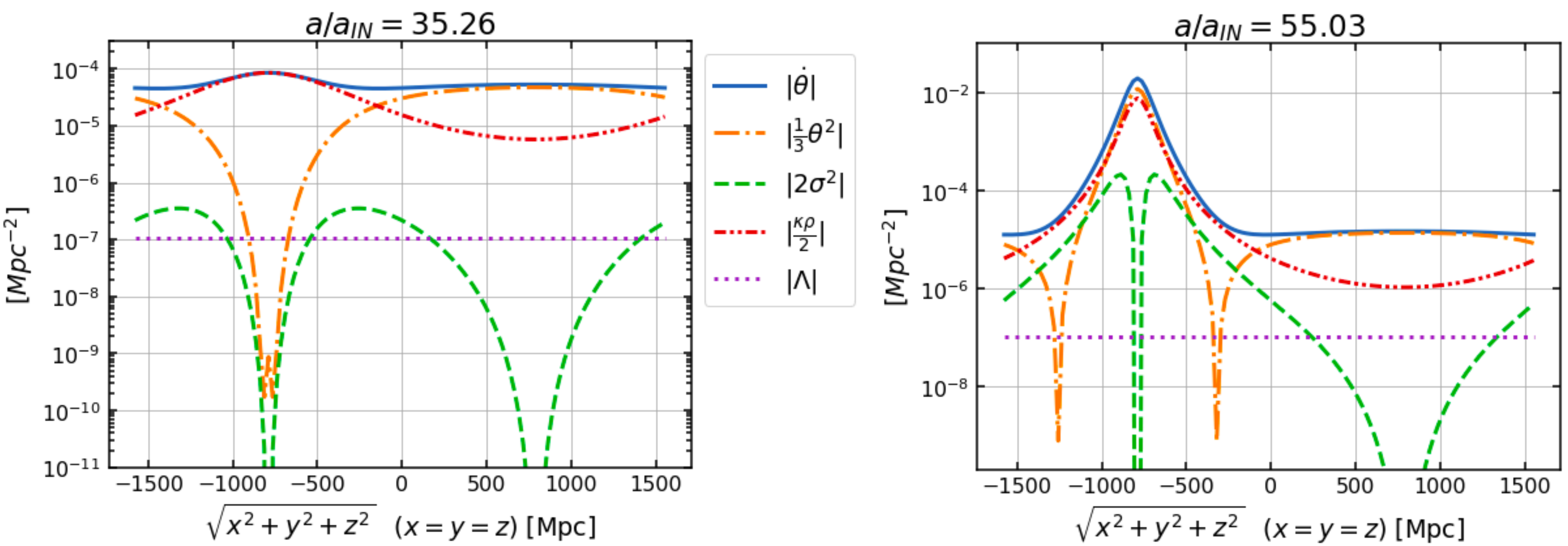


At large radii, the spherical collapse model is no longer a good fit as it overestimates collapse time.

Contributions to the Raychaudhuri equation

$$\dot{\Theta} + \frac{1}{3}\Theta^2 + 2\sigma^2 + \frac{\kappa\rho}{2} - \Lambda = 0$$

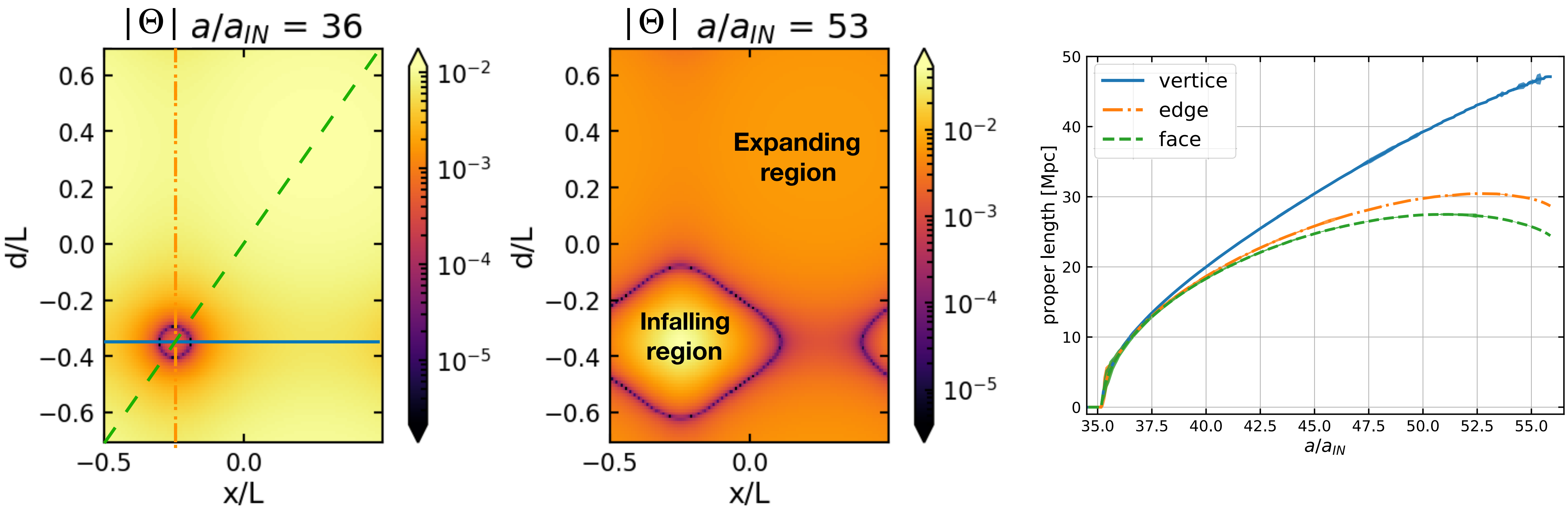
At Turn around — Right before the crash



Θ expansion, σ^2 shear, $\kappa = 8\pi$, ρ energy density, Λ cosmological constant

Collapse evolution

Turn-around boundary $\Theta = 0$

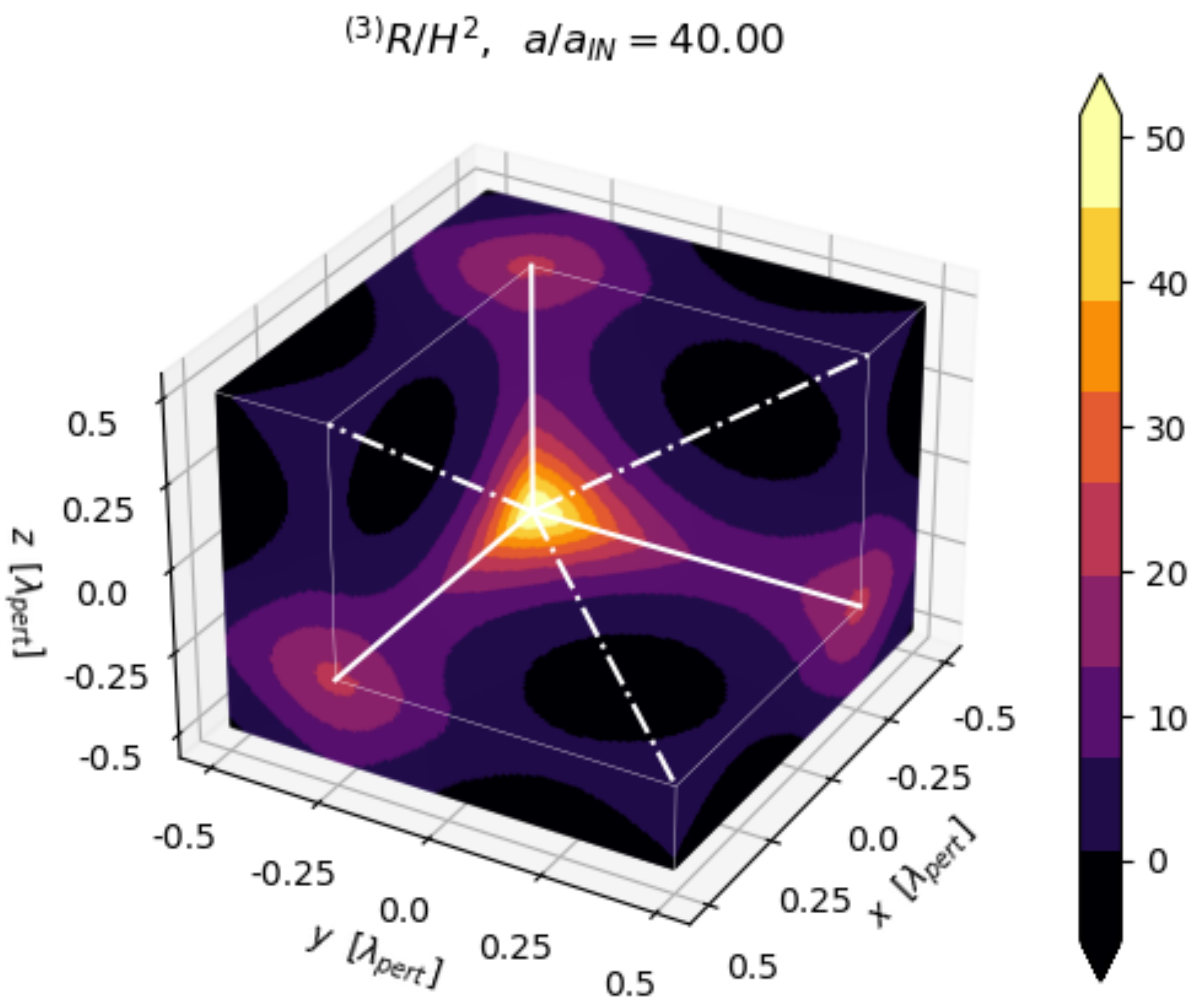


The expansion of the turn-around boundary depends on the initial distribution.

The directions going through under-dense regions eventually stop expanding their infalling region and reduce in size.

Gravito-electromagnetism with EBWeyl

Riemann tensor: $R_{\alpha\beta\mu\nu}$ *Trace* → **Ricci Tensor:** $R_{\beta\nu}$ *Trace* → **Ricci Scalar:** R



Gravito-electromagnetism with EBWeyl

Riemann tensor: $R_{\alpha\beta\mu\nu}$ → *Traceless part* → **Weyl tensor:** $C_{\alpha\beta\mu\nu}$ → *Projection along the fluid*

Electric part: $E_{\alpha\mu} = u^\beta u^\nu C_{\alpha\beta\mu\nu}$

Magnetic part: $B_{\beta\nu} = u^\alpha u^\mu C^*_{\alpha\beta\mu\nu}$

❖ **Gauge invariant at 1st order**

The Stewart-Walker lemma states that, at 1st order, perturbations of covariant quantities are gauge invariant if their background value vanishes.

J.M.Stewart and M.Walker (1974)

An FLRW universe is conformally flat, so $E_{\alpha\beta} = B_{\alpha\beta} = 0$, therefore they are gauge invariant at 1st order.

However $E_{\alpha\beta}$ and $B_{\alpha\beta}$ are frame dependent so we consider the comoving frame.

❖ **Physically meaningful** With the Weyl scalars $\Psi_{0...4}$

Gravitational pull | **Frame Dragging** | **Gravitational Waves**

$$E^{\alpha\beta} = \Re(\Psi_2)e_C^{\alpha\beta} + \frac{1}{2}\Re(\Psi_0+\Psi_4)e_{T+}^{\alpha\beta} + \frac{1}{2}\Im(\Psi_0-\Psi_4)e_{T\times}^{\alpha\beta} - 2\Re(\Psi_1-\Psi_3)e_1^{(\alpha}e_2^{\beta)} - 2\Im(\Psi_1+\Psi_3)e_1^{(\alpha}e_3^{\beta)} \simeq \frac{1}{2}D^\alpha D^\beta(\Psi+\Phi)$$

$$B^{\alpha\beta} = -\Im(\Psi_2)e_C^{\alpha\beta} - \frac{1}{2}\Im(\Psi_0+\Psi_4)e_{T+}^{\alpha\beta} + \frac{1}{2}\Re(\Psi_0-\Psi_4)e_{T\times}^{\alpha\beta} + 2\Im(\Psi_1-\Psi_3)e_1^{(\alpha}e_2^{\beta)} - 2\Re(\Psi_1+\Psi_3)e_1^{(\alpha}e_3^{\beta)} \simeq 0$$

At first order with only scalar perturbations

Ψ and Φ the Bardeen potentials

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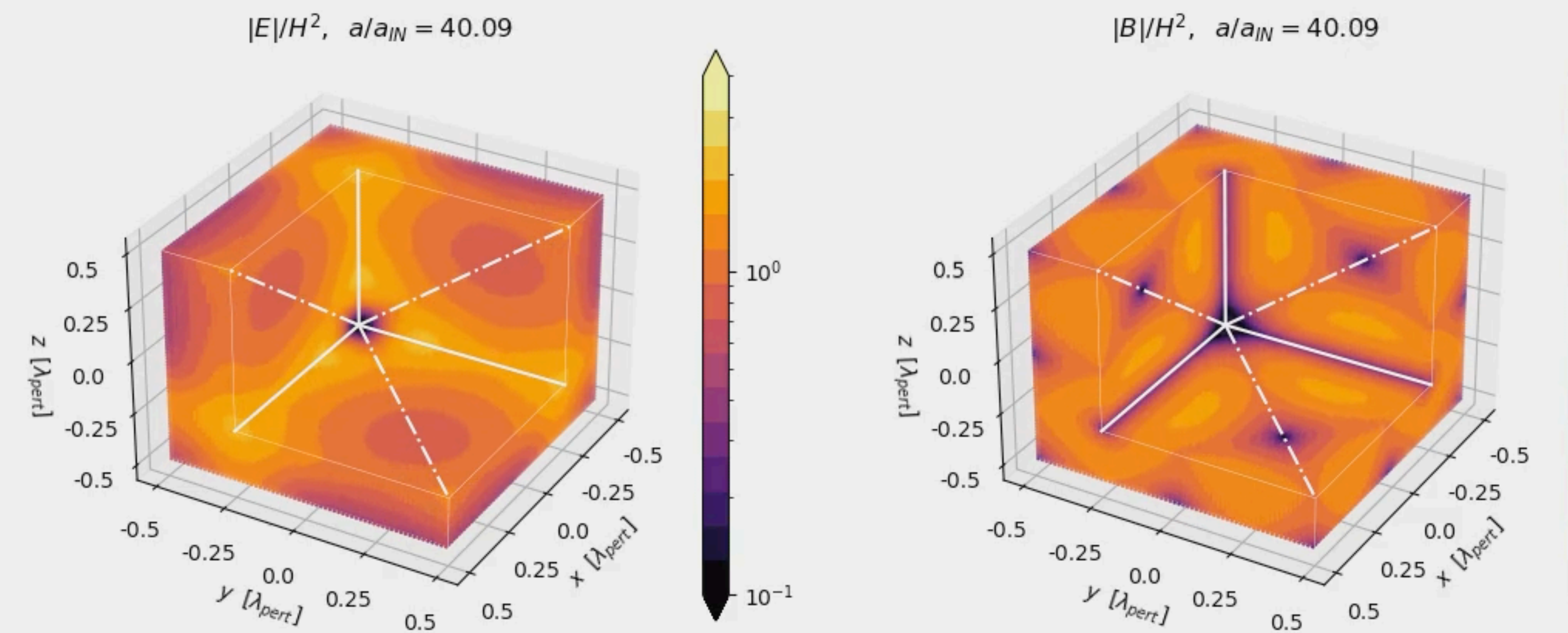
Electric part:

$$E_{\alpha\mu} = u^\beta u^\nu C_{\alpha\beta\mu\nu}$$

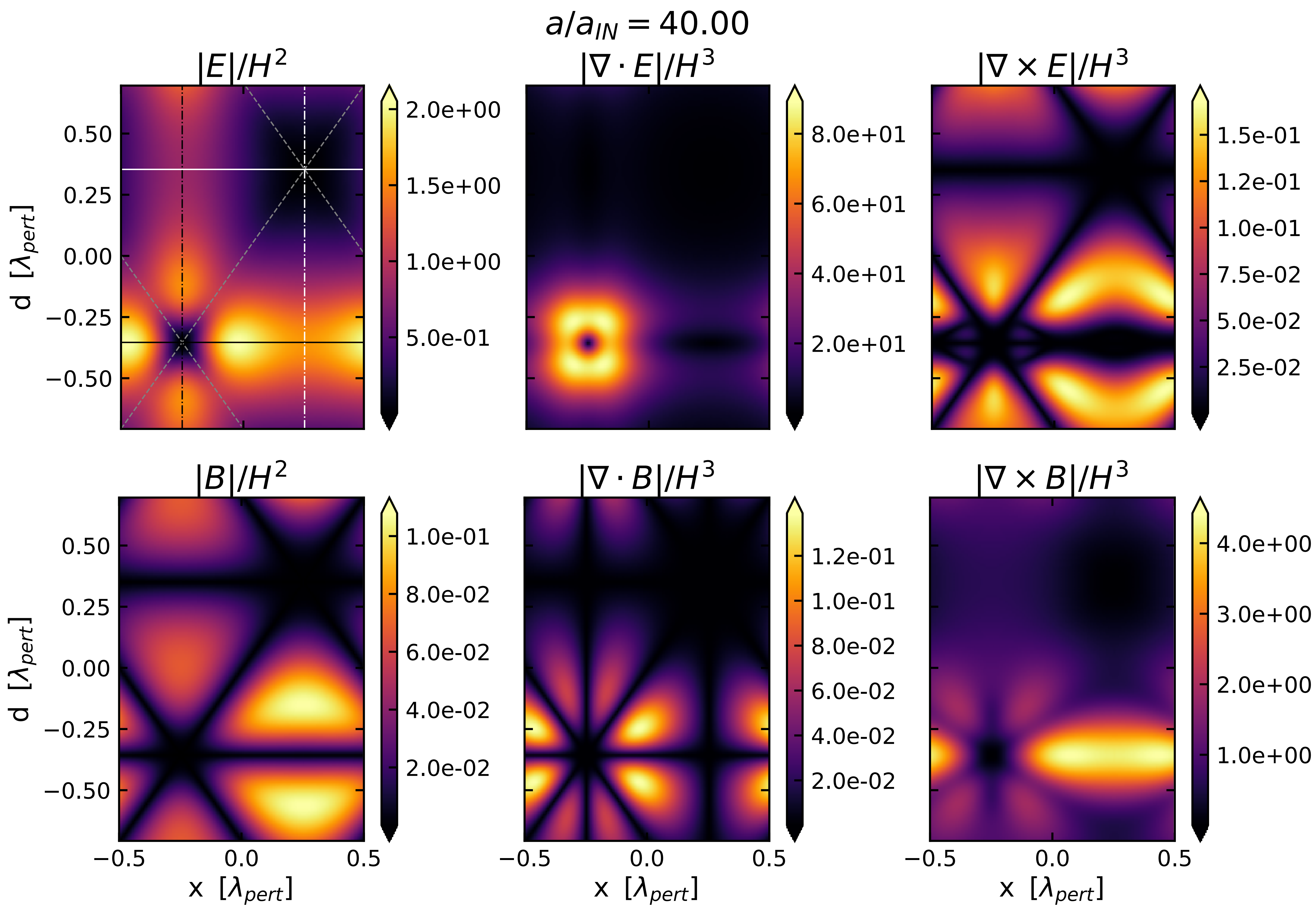
Magnetic part:

$$B_{\beta\nu} = u^\alpha u^\mu C^*_{\alpha\beta\mu\nu}$$

$$|E| = \sqrt{E_{\alpha\beta}E^{\alpha\beta}}$$



Gravito-electromagnetism with EBWeyl



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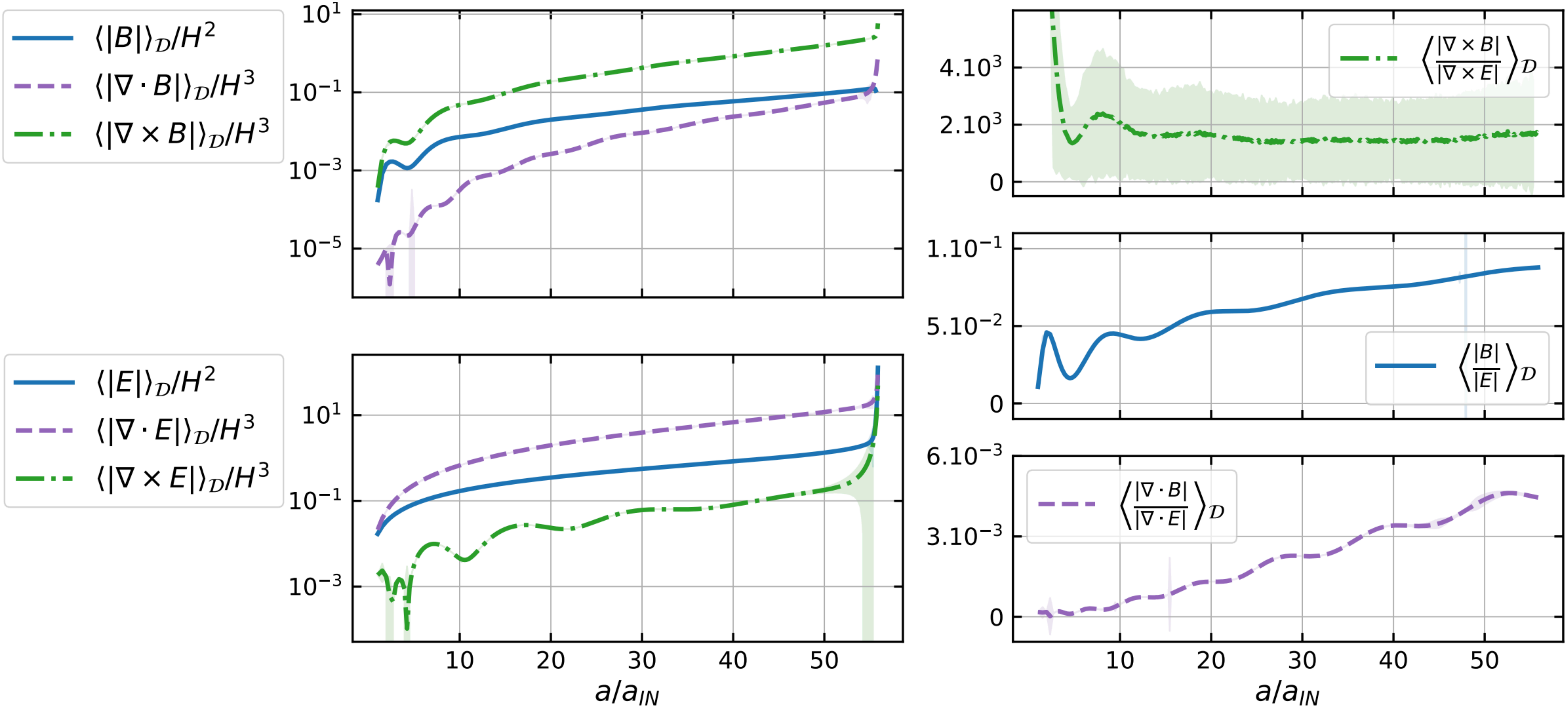


" in type-N and III spacetimes

$$E_{\alpha\beta}E^{\alpha\beta} = B_{\alpha\beta}B^{\alpha\beta} > 0 \text{ "}$$

The electric and magnetic Weyl tensors,

W.B.Bonnor

(Class. Quantum Gravity 1995)

Classification of Petrov type with EBWeyl

Riemann tensor: $R_{\alpha\beta\mu\nu}$ → *Traceless part* → **Weyl tensor:** $C_{\alpha\beta\mu\nu}$

Projection along a null tetrad basis

Weyl Scalars

Newmann-Penrose formalism

$$\Psi_0 = C_{\alpha\beta\mu\nu} l^\alpha m^\beta l^\mu m^\nu$$

$$\Psi_1 = C_{\alpha\beta\mu\nu} l^\alpha k^\beta l^\mu m^\nu$$

$$\Psi_2 = C_{\alpha\beta\mu\nu} l^\alpha m^\beta \overline{m}^\mu k^\nu$$

$$\Psi_3 = C_{\alpha\beta\mu\nu} l^\alpha k^\beta \overline{m}^\mu k^\nu$$

$$\Psi_4 = C_{\alpha\beta\mu\nu} k^\alpha \overline{m}^\beta k^\mu \overline{m}^\nu$$

With $(l^\mu, k^\mu, m^\mu, \bar{m}^\mu)$ an arbitrary null tetrad basis

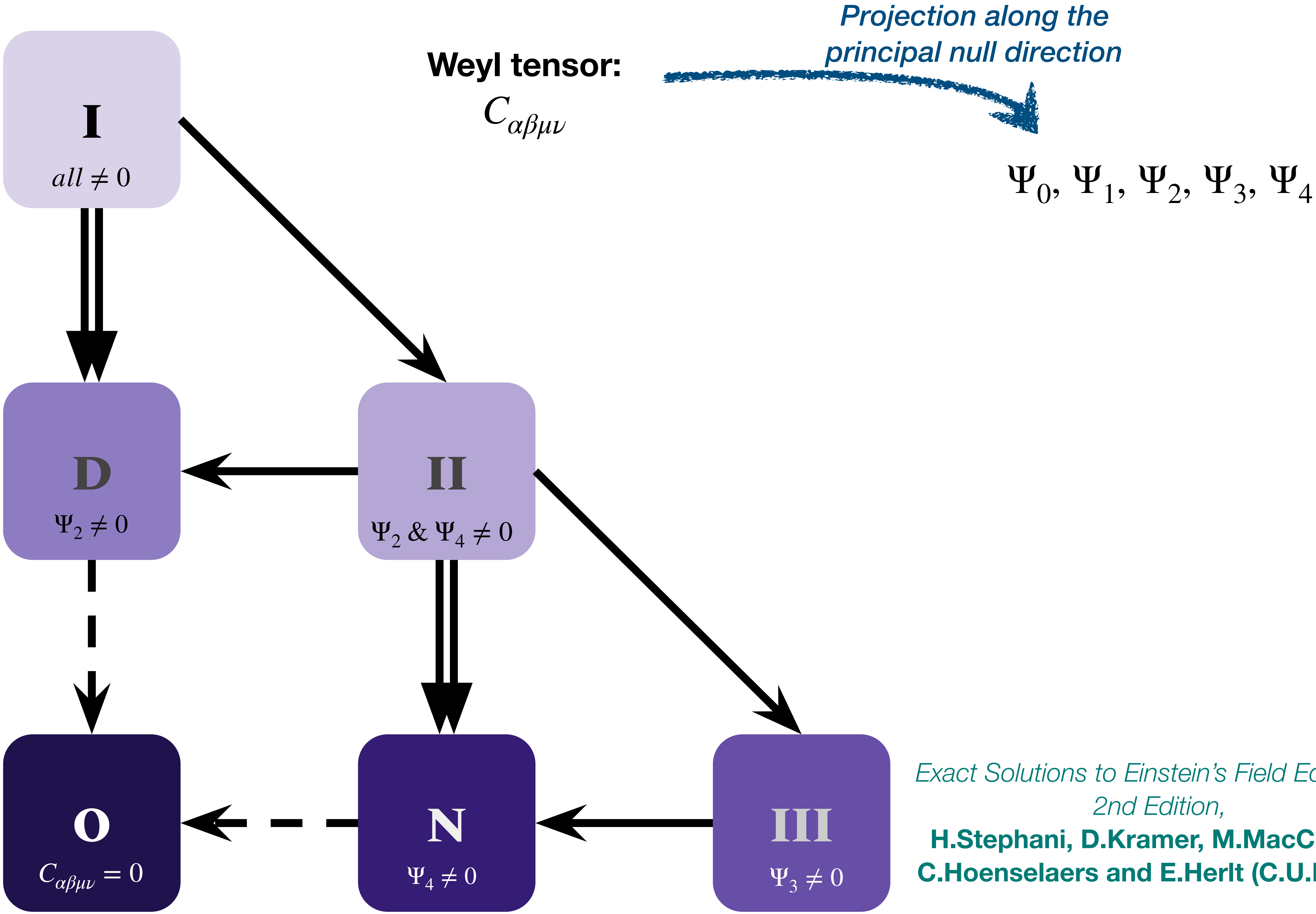
Rotate the tetrad frame to minimise the number of non-zero Well scalars
principal null direction

Need to solve 4th-order complex polynomial

Instead, compute the discriminant
Construct the complex scalar invariants
I, J, K, L, N

Exact Solutions to Einstein's Field Equations 2nd Edition,
H.Stephani, D.Kramer, M.MacCallum, C.Hoenselaers and E.Herlt (C.U.P. 2003)

Classification of Petrov type with EBWeyl



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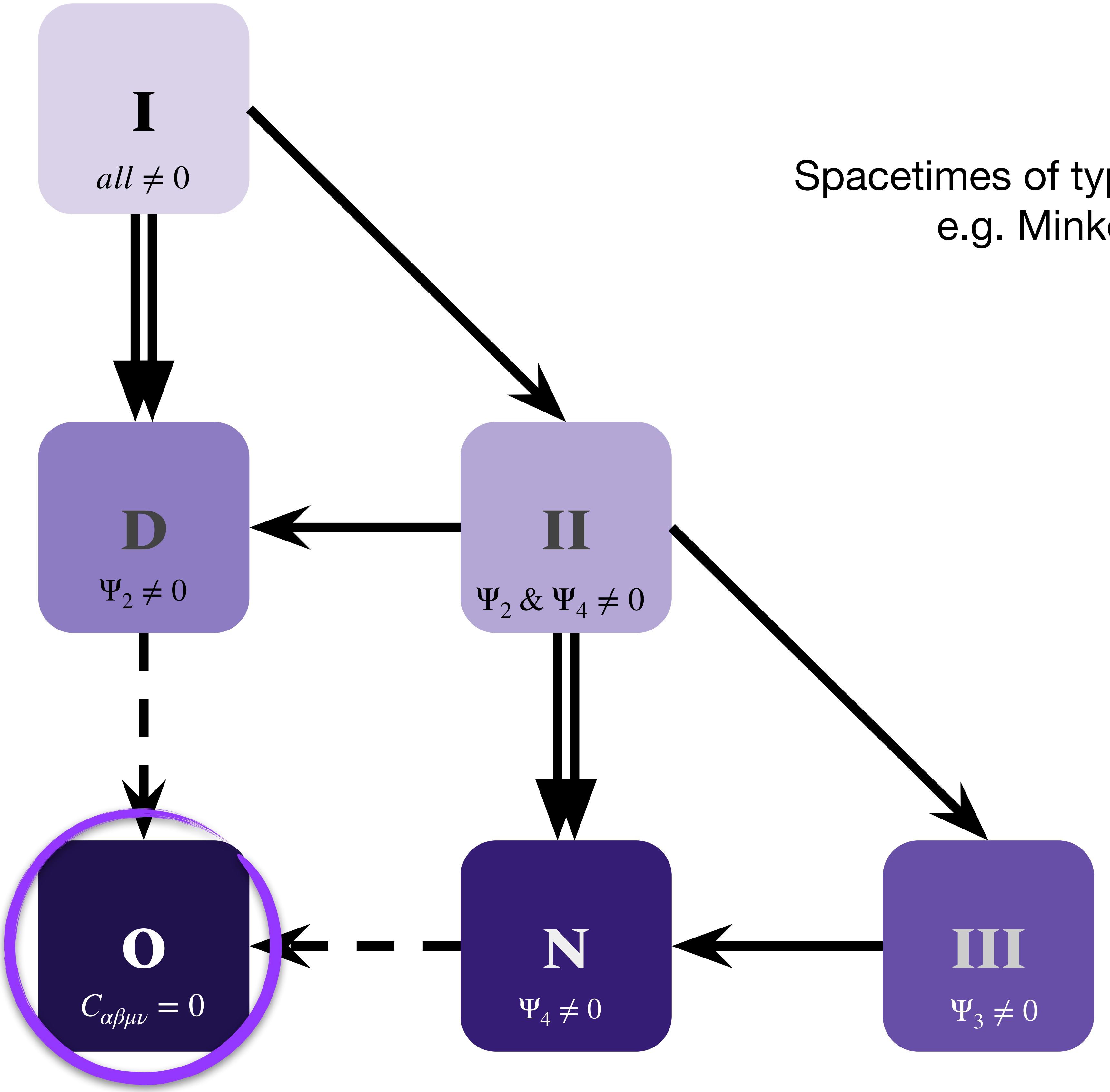

Spacetimes of type O are conformally flat, e.g. Minkowski and FLRW.

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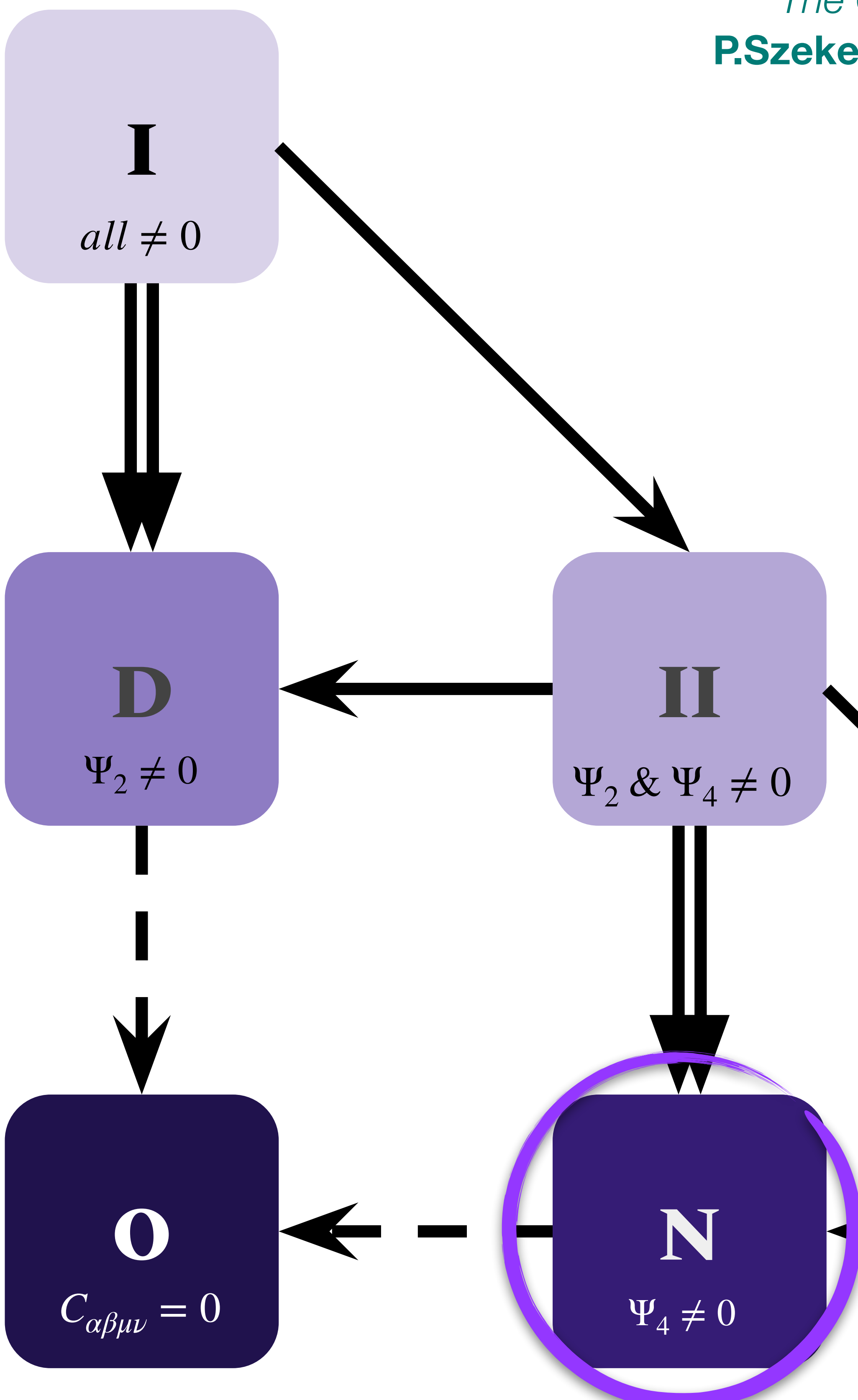
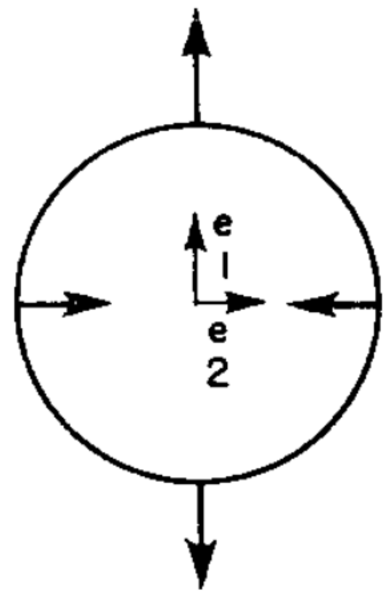
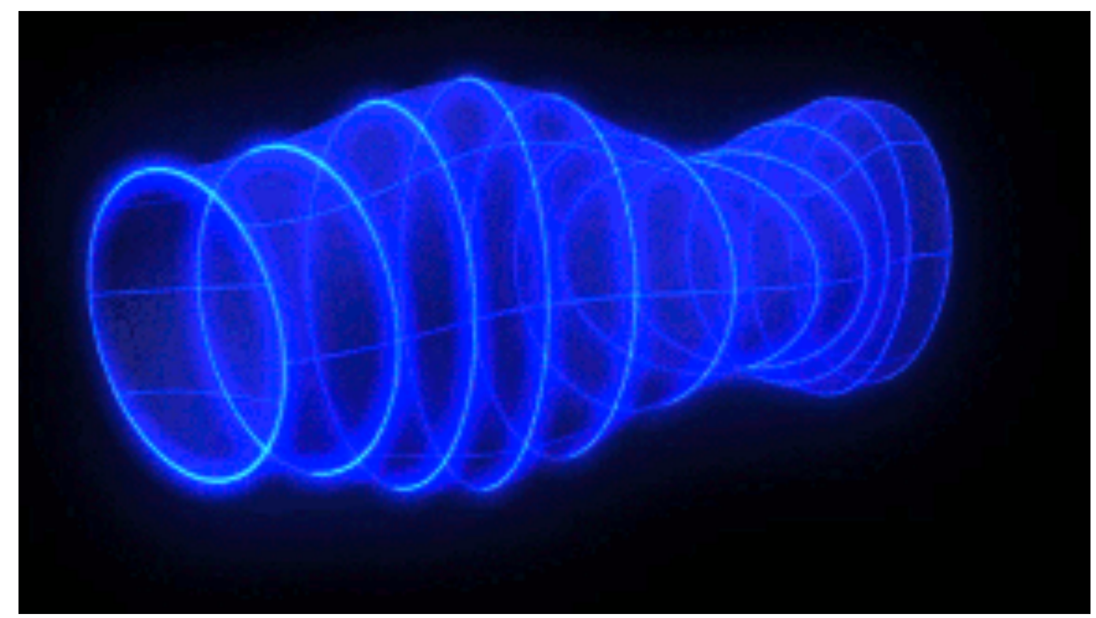


The Gravitational Compass,
P.Szekeres (J. Math. Phys. 1965)



Ψ_4 generates a transverse geodesic deviation

Spacetimes with pure transverse gravitational waves



Credit: ESA–C.Carreau

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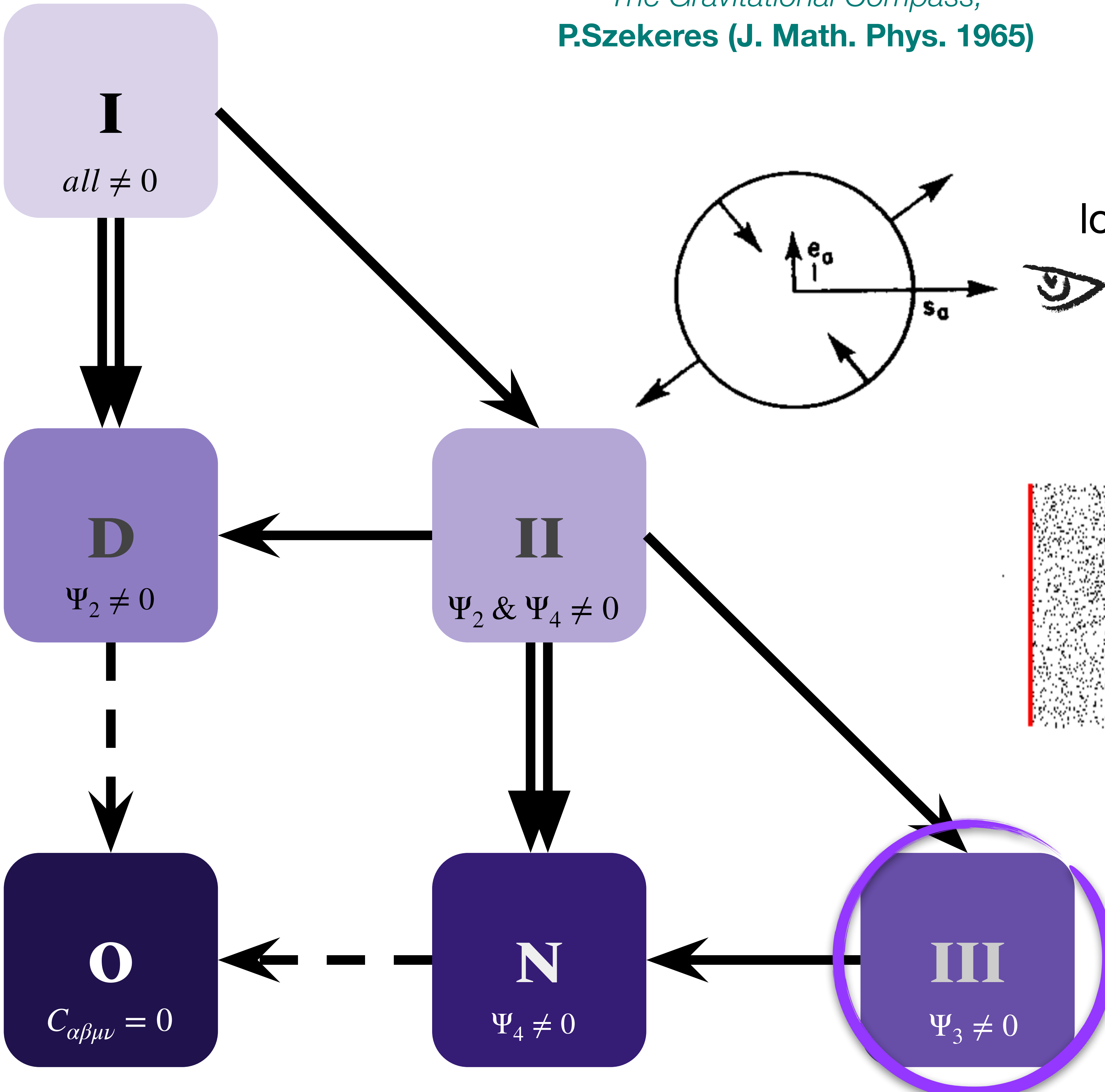
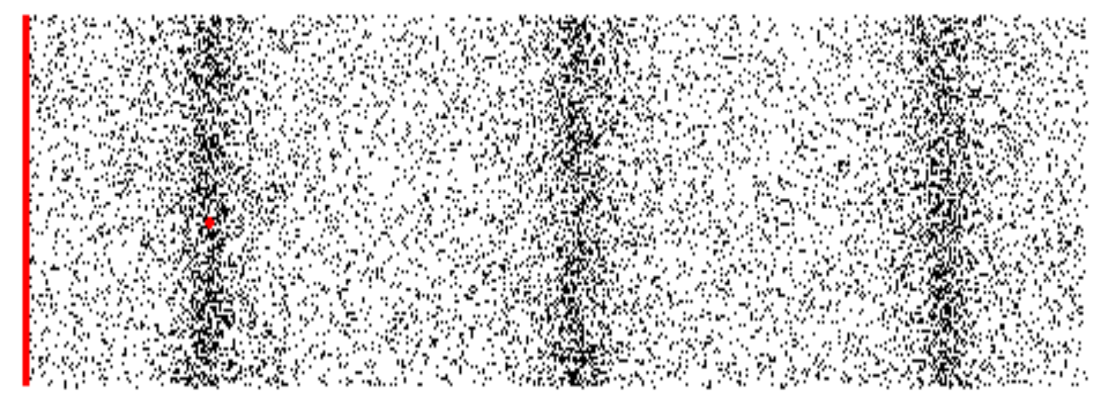


The Gravitational Compass,
P.Szekeres (J. Math. Phys. 1965)

Ψ_3 generates a longitudinal tidal distortion

Spacetimes with pure longitudinal gravitational waves



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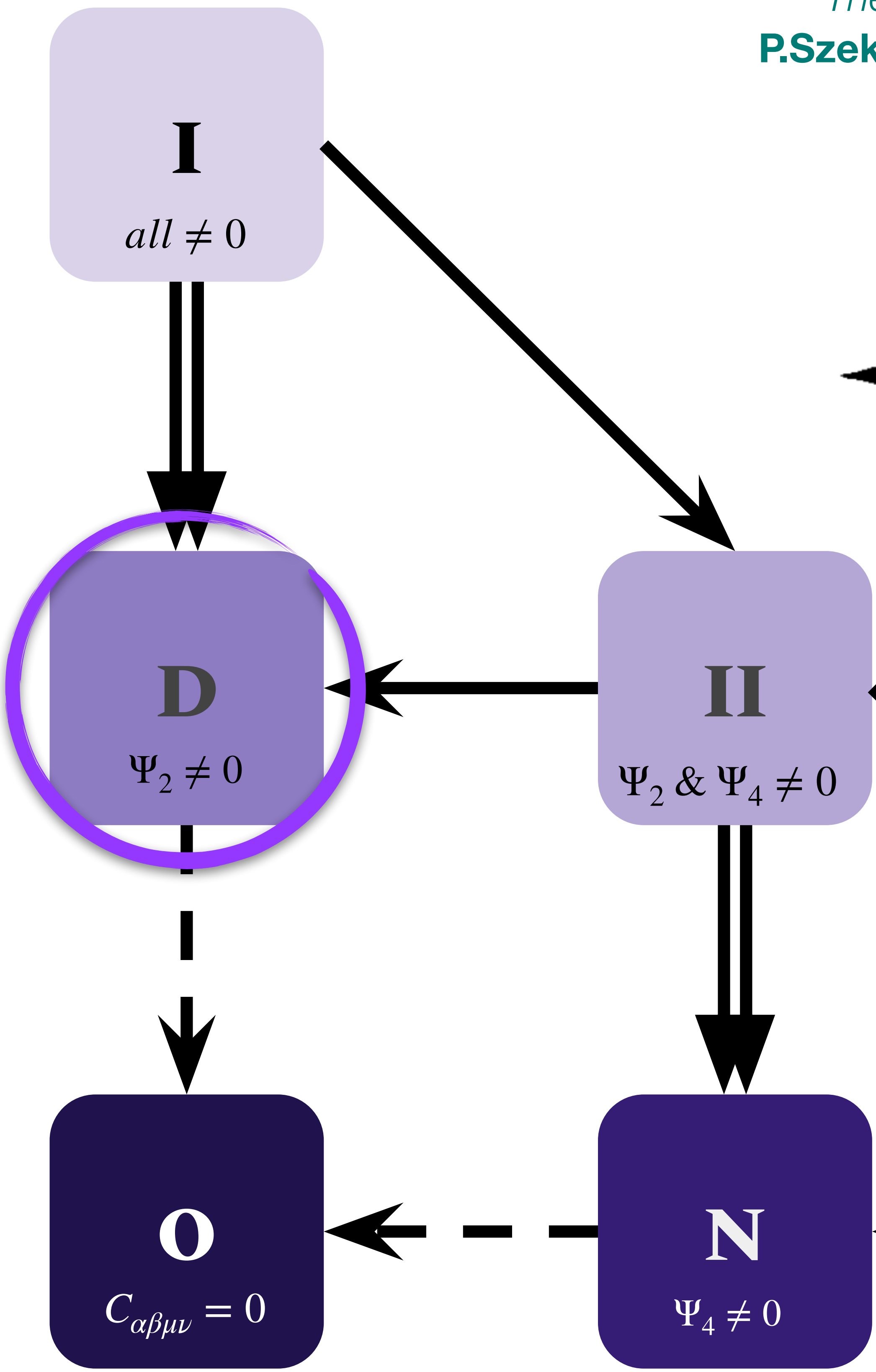

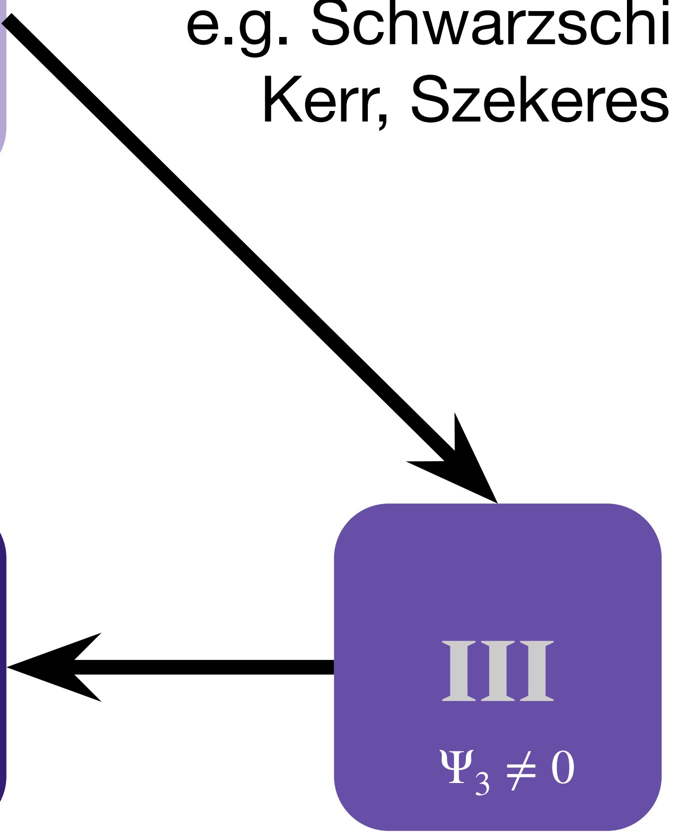

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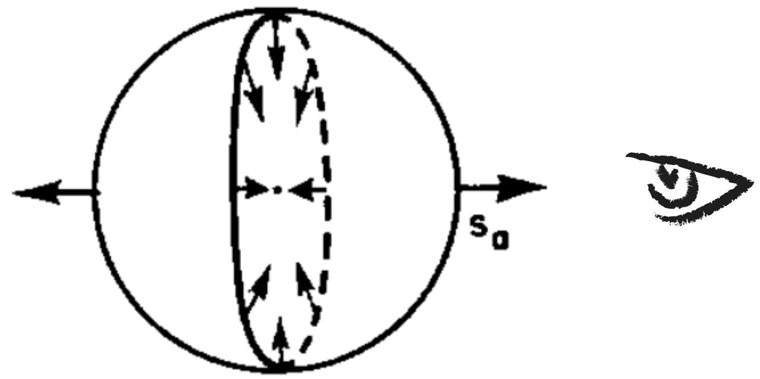
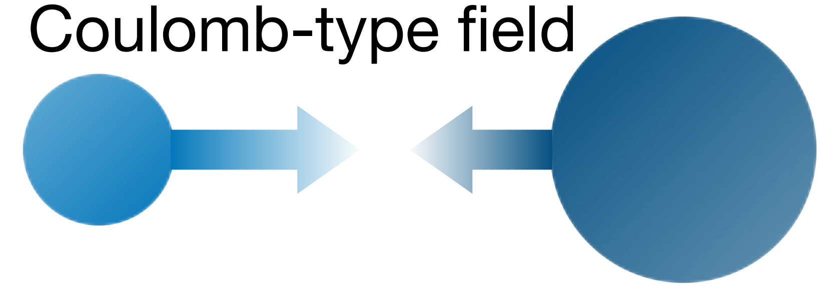
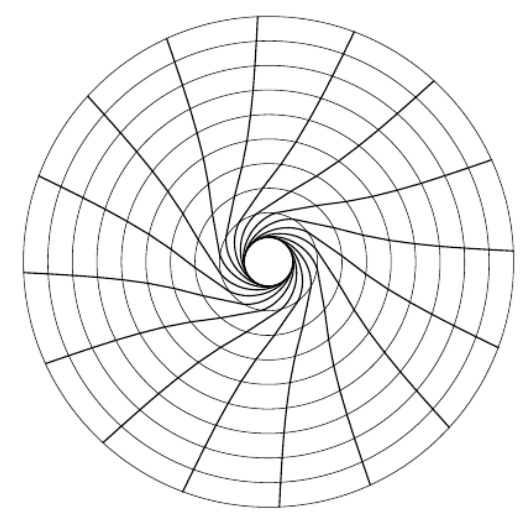


$\Re(\Psi_2)$ generates a tidal distortion associated with a Coulomb-type field



$\Im(\Psi_2)$ associated with frame dragging



e.g. Schwarzschild, Kerr, Szekeres

Credit: Michael Cramer Andersen

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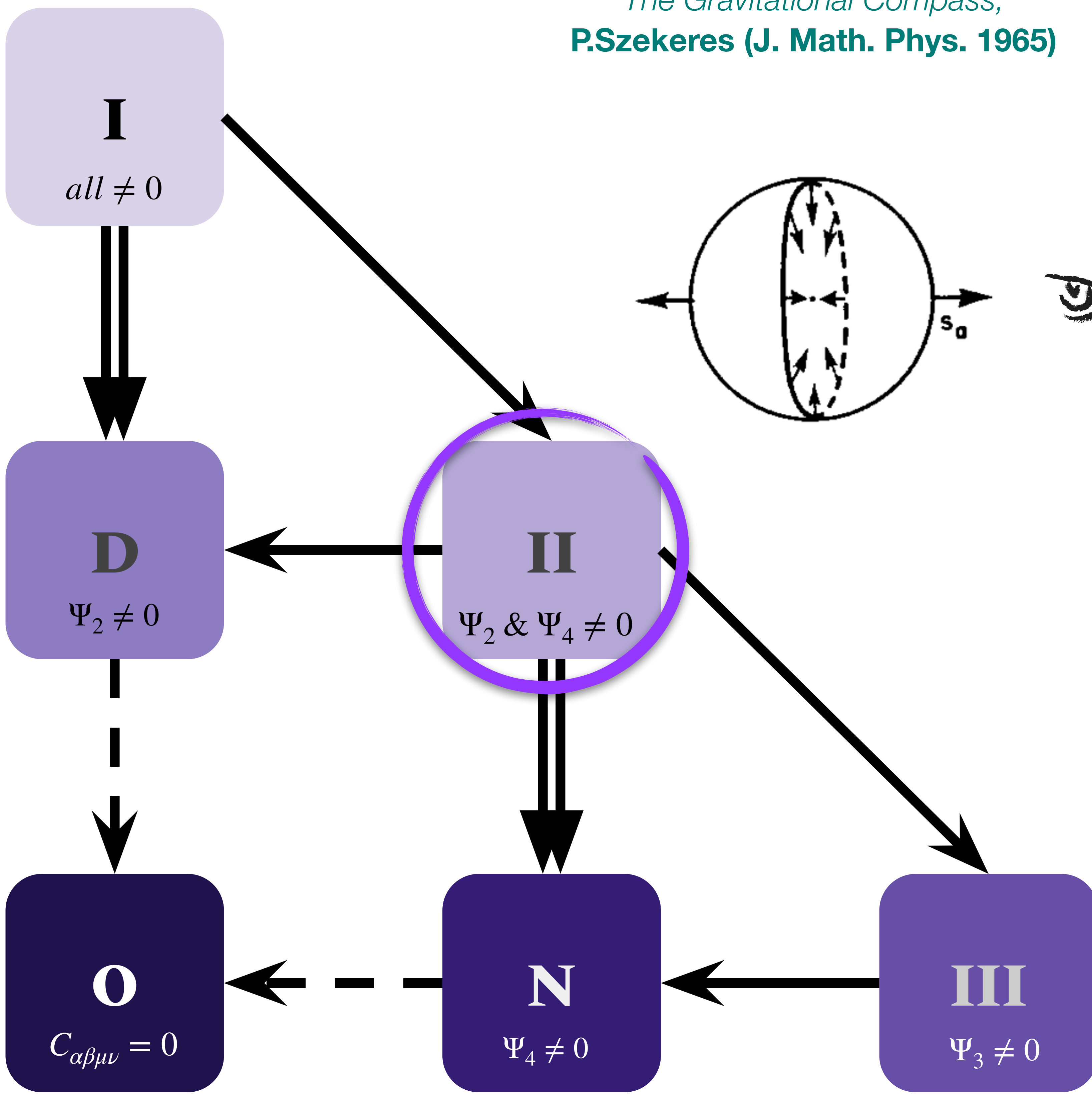

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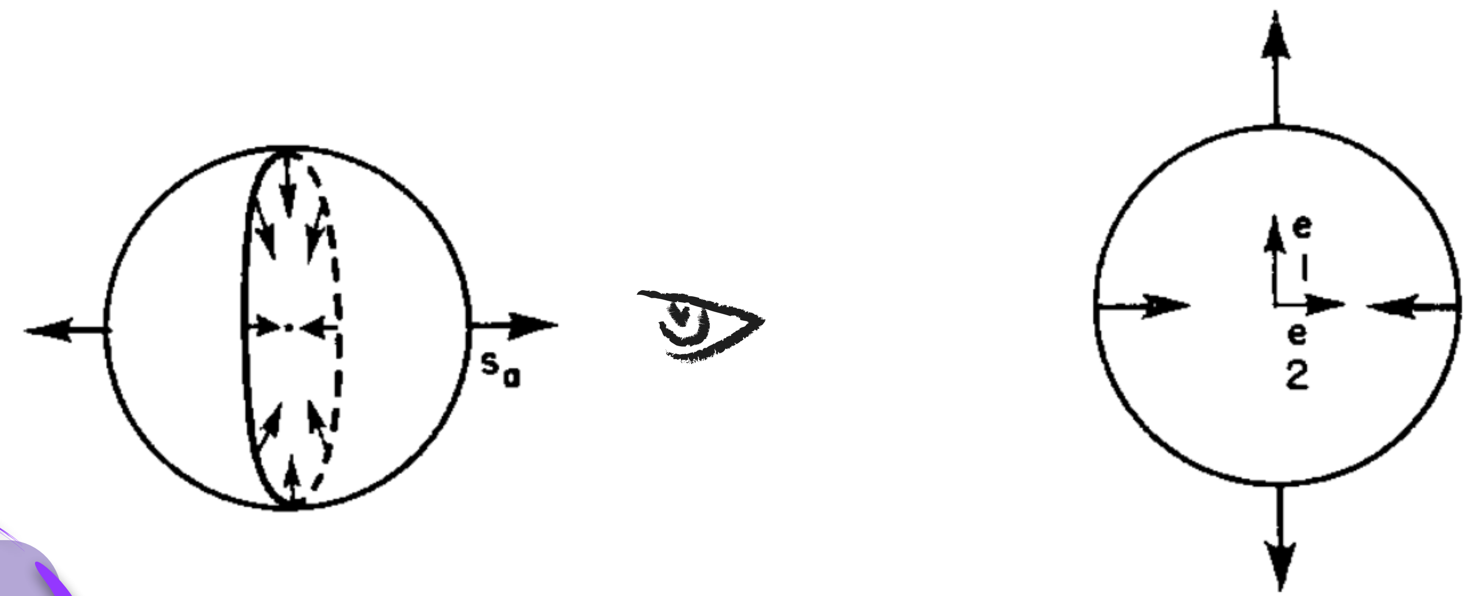

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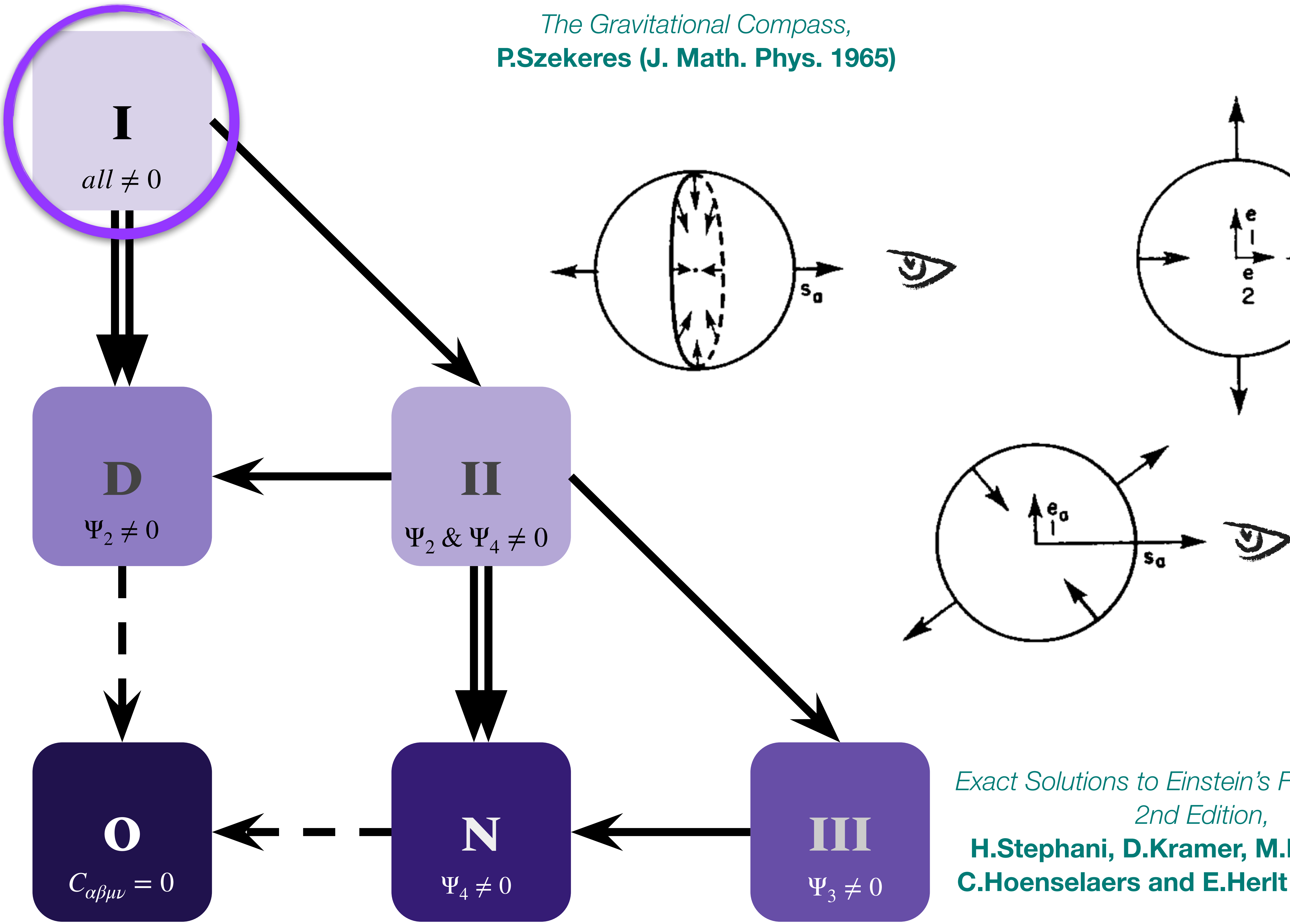

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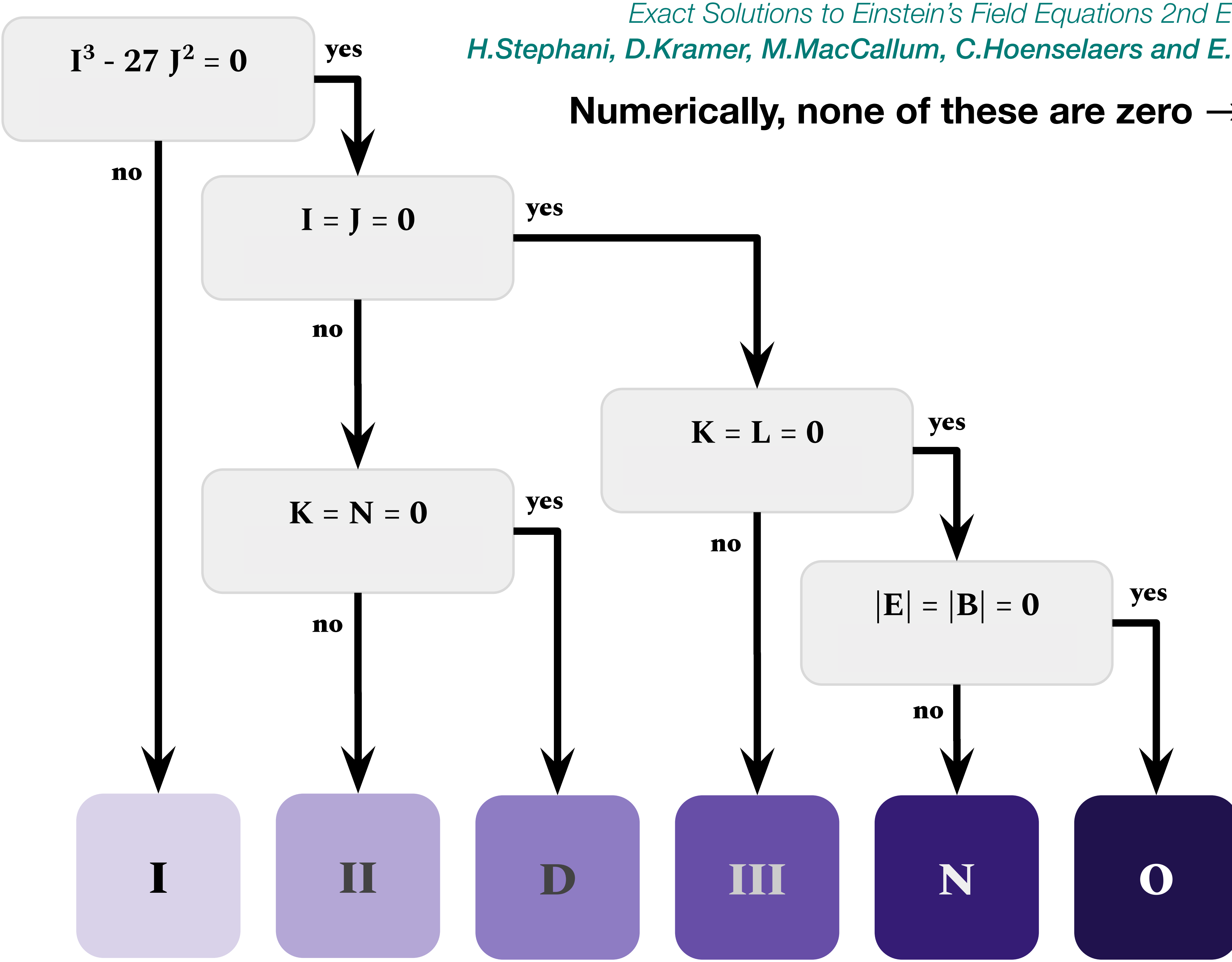
Numerically, none of these are zero → type I everywhere



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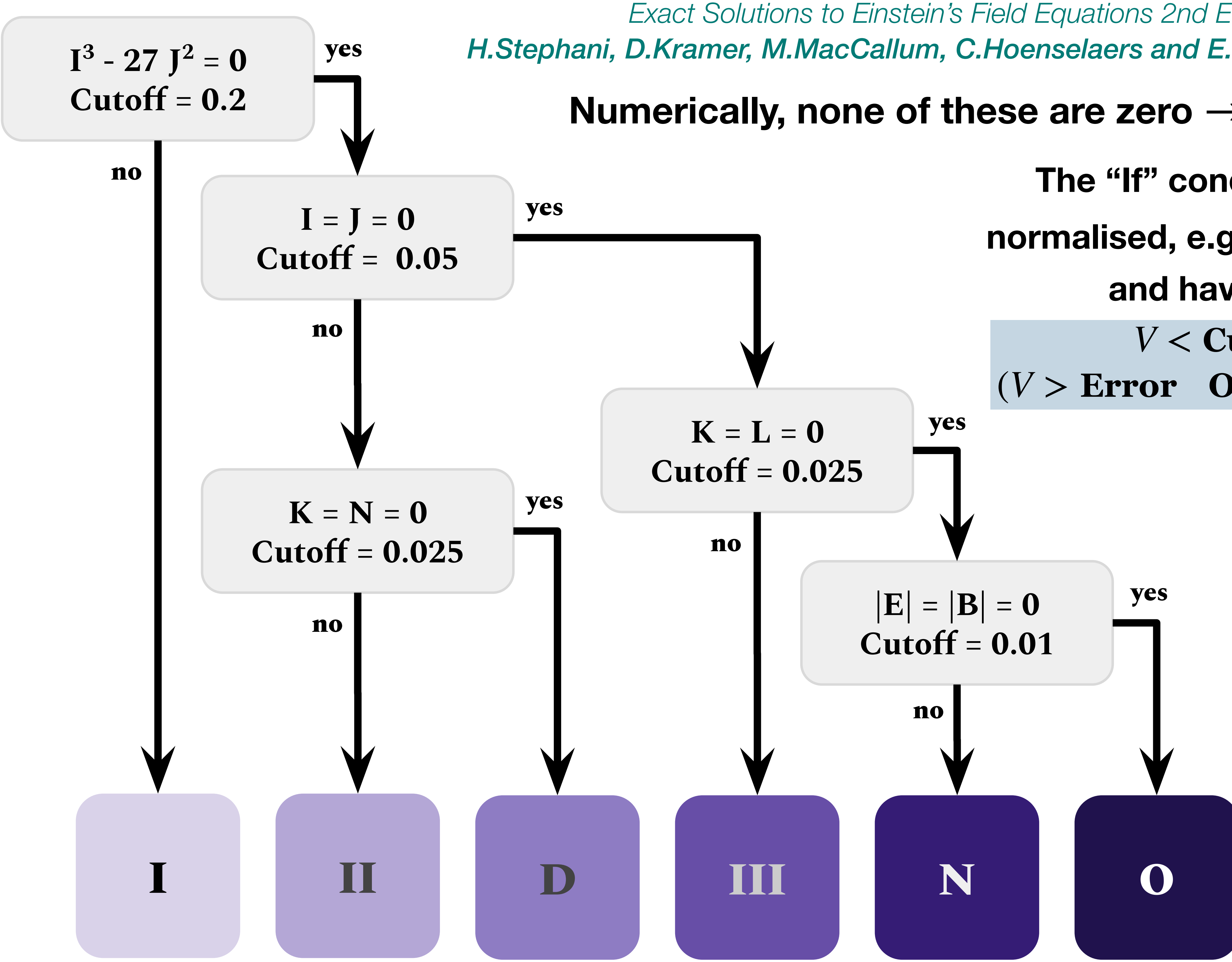
Numerically, none of these are zero → type I everywhere

The "If" condition needs to be normalised, e.g. : $V = |Re(\mathbf{I}^{1/2})|/H^2$ **and have a threshold:**

$$V < \textbf{Cutoff} \quad \textbf{AND}$$
$$(V > \textbf{Error} \quad \textbf{OR} \quad \textbf{Cutoff} > \textbf{Error})$$



Classification of Petrov type with EBWeyl

- Transition from

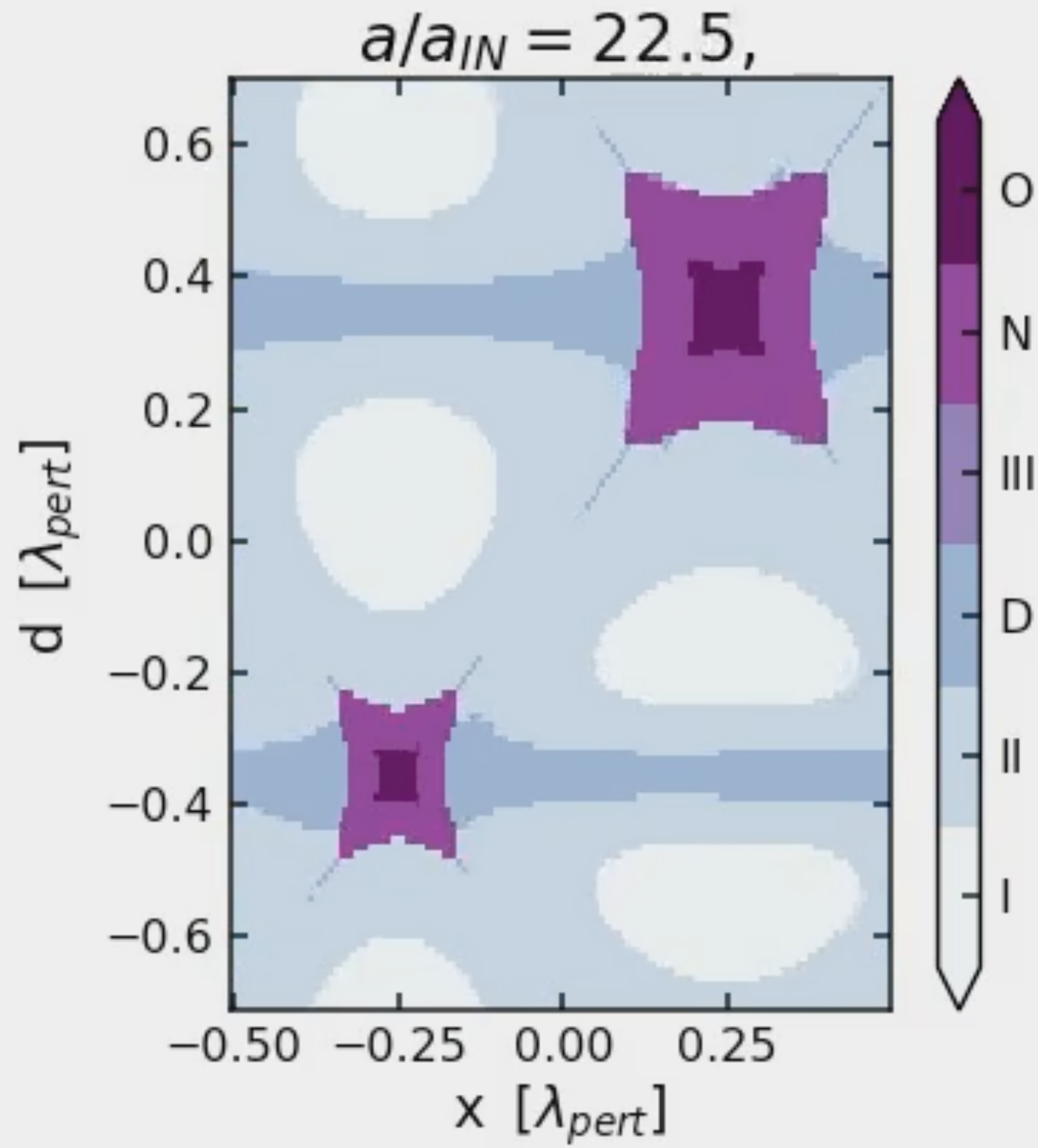
  $$\mathbf{O} \rightarrow \mathbf{N} \rightarrow \mathbf{D} \rightarrow \mathbf{II} \rightarrow \mathbf{I}.$$

- Strong presence of type $\mathbf{N}$, that of gravitational wave spacetimes.

- In the very centre of the over-density, it is type $\mathbf{O}$. This is consistent with the spherical collapse model.

- Mostly $\mathbf{D}$ along the filaments.

- $\mathbf{O}$ remains in the under-density as it is conformally flat.



Conclusion

❖ At the peak of the over-density, the spherical collapse model is an excellent approximation. This is because we find that the shear is locally negligible. Then, neglecting the shear in the Raychaudhuri equation gives the spherical collapse model.

❖ The spacetime is of Petrov type **I**, however the leading order type transitions from a special to general spacetime with notably a strong presence of type **N**.

❖ We have type **O** in the under-density and its surrounding region, and in the centre of the over-density. This is in line with the spherical collapse model.

❖ The electric part of the Weyl tensor is strongest along the filaments, with significant divergence. The magnetic part is strongest around the filaments, with significant curl. Type **D** is predominant along the filaments.

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Future ventures:

- Study the evolution of regions of 10^{12}-10^{16} Msun that include anisotropy & deviation from spherical symmetry
- Different initial curvature perturbation, mode coupling, extend ICPertFLRW
- Different gauges and characterise gauge invariant variables